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by

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SOME THEOREMS ON COLOCALLY CONNECTED CONTINUA

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Abstract. We show that each refinable map preserves colocal connectedness of the domain while a proximately refinable map does not necessarily. Also, we prove that colocal connectedness is a Whitney property and is not a Whitney reversible property.

1. Introduction

In this paper, unless otherwise stated, all spaces are assumed to be metrizable. When we use the term "function," we do not assume it to be continuous necessarily, while we require "map" to be continuous. Let X be a continuum and $x \in X$. We say that X is *colocally connected* if, for each $x \in X$ and each neighborhood $V \subseteq X$ of x, there exists an open neighborhood $U \subseteq X$ of x such that $x \in U \subseteq V$ and $X \setminus U$ is connected. A continuum X is said to be *aposyndetic* if, for any two distinct points $x, y \in X$, there exists a subcontinuum $T \subseteq X$ such that $x \in \mathrm{Int}_X T \subseteq T \subseteq X \setminus \{y\}$, where $\mathrm{Int}_X T$ denotes the interior of T in X. F. Burton Jones introduced aposyndetic continua in [10]. Since then, aposyndetic continua have been studied for many years. It is known that every colocally connected continuum is aposyndetic [12, Remark 5.4.15]. As a consequence, colocal connectedness implies many properties of continua. (See [3, Figure 6] and also [2, p. 239] for other properties derived from colocal connectedness.)

If (X, d) is a space and $A \subseteq X$, then we denote $\sup\{d(a, b) \mid a, b \in A\}$ by $\mathrm{diam}_d A$. Let (X, d_X) and (Y, d_Y) be continua and let $\varepsilon > 0$. A surjective

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