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by

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EXAMPLES OF STRONGLY RIGID COUNTABLE (SEMI)HAUSDORFF SPACES

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Abstract. A topological space X is *strongly rigid* if each non-constant continuous map $f : X \to X$ is the identity map of X. A Hausdorff topological space X is called *Brown* if for any nonempty open sets $U, V \subseteq X$ the intersection $\overline{U} \cap \overline{V}$ is infinite. We prove that every second-countable Brown Hausdorff space X admits a stronger topology $\mathcal{T}'$ such that $X' = (X, \mathcal{T}')$ is a strongly rigid Brown space. This construction yields an example of a countable anticompact Hausdorff space X which is strongly rigid. By the same method, we construct a strongly rigid semi-Hausdorff k-metrizable space containing a non-closed compact subset.

1. Introduction

A topological space X is called

- *rigid* if every homeomorphism $f : X \to X$ coincides with the identity map of X;
- *strongly rigid* if every non-identity continuous map $f : X \to X$ is constant.

Proposition 1.1. *Every strongly rigid space X is connected.*

Proof. Assuming that X is disconnected, we can write X as the union $X = U \cup V$ of two disjoint nonempty open sets U and V. Choose any points $u \in U$ and $v \in V$ and consider the continuous map $f : X \to \{u, v\}$ such that $f^{-1}(u) = V$ and $f^{-1}(v) = U$. This map witnesses that the space X is not strongly rigid. □

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