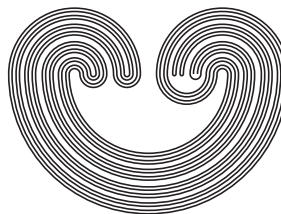


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## EXAMPLES OF STRONGLY RIGID COUNTABLE (SEMI)HAUSDORFF SPACES

by

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## EXAMPLES OF STRONGLY RIGID COUNTABLE (SEMI)HAUSDORFF SPACES

TARAS BANAKH AND YARYNA STELMAKH

**ABSTRACT.** A topological space  $X$  is *strongly rigid* if each non-constant continuous map  $f : X \rightarrow X$  is the identity map of  $X$ . A Hausdorff topological space  $X$  is called *Brown* if for any nonempty open sets  $U, V \subseteq X$  the intersection  $\overline{U} \cap \overline{V}$  is infinite. We prove that every second-countable Brown Hausdorff space  $X$  admits a stronger topology  $\mathcal{T}'$  such that  $X' = (X, \mathcal{T}')$  is a strongly rigid Brown space. This construction yields an example of a countable anticomcompact Hausdorff space  $X$  which is strongly rigid. By the same method, we construct a strongly rigid semi-Hausdorff  $k$ -metrizable space containing a non-closed compact subset.

### 1. INTRODUCTION

A topological space  $X$  is called

- *rigid* if every homeomorphism  $f : X \rightarrow X$  coincides with the identity map of  $X$ ;
- *strongly rigid* if every non-identity continuous map  $f : X \rightarrow X$  is constant.

**Proposition 1.1.** *Every strongly rigid space  $X$  is connected.*

*Proof.* Assuming that  $X$  is disconnected, we can write  $X$  as the union  $X = U \cup V$  of two disjoint nonempty open sets  $U$  and  $V$ . Choose any points  $u \in U$  and  $v \in V$  and consider the continuous map  $f : X \rightarrow \{u, v\}$  such that  $f^{-1}(u) = V$  and  $f^{-1}(v) = U$ . This map witnesses that the space  $X$  is not strongly rigid.  $\square$

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*Key words and phrases.* anticomcompact spaces, Brown space,  $k$ -metrizable space, rigid space, Rudin–Keisler incomparable ultrafilters, strongly rigid space.

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