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SOME REALCOMPACT SPACES

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Abstract. We present examples of realcompact spaces with closed subsets that are C^*-embedded but not C-embedded, including one where the closed set is a copy of $\mathbb{N}$.

Introduction

The purpose of this note is to provide some examples of realcompact (but not compact) spaces that have closed subspaces that are C^*-embedded but not C-embedded, and, in particular, an example where the closed subspace is a copy of the discrete space $\mathbb{N}$ of natural numbers—what we henceforth call *a closed copy of* $\mathbb{N}$.

The reason for our interest is that we are not aware of any such examples. For instance, the examples in [5] of C^*- but not C-embedded subsets are not all closed and, when they are closed, the pseudocompactness of the ambient space makes C-embedding impossible.

The only explicit question of this nature that we could find is in [7, Question 1], which asks whether C^*-embedded subsets (not necessarily closed) of first-countable spaces are C-embedded. In that case, there is an independence result: There is a counterexample if $\mathfrak{b} = \mathfrak{s} = \mathfrak{c}$, and, in the model obtained by adding supercompact many random reals, the implication holds; see [1].

The more specific question of having a closed copy of $\mathbb{N}$, that is, C^*-embedded but not C-embedded, arises from an analysis of their position in powers of the real line; see section 2 for an explanation.

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