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by

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CHARACTERIZATION OF ARCS BY PRODUCTS AND DIAGONALS

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Abstract. We prove that a continuum X is a Hausdorff arc if, and only if, $X^2 - D$ is not connected, where D is the diagonal of X^2.

Let us begin with a definition.

Definition 1. A *continuum* is a compact connected Hausdorff space. Let X be a continuum. A point $p \in X$ is a *cut point* of X if $X - \{p\}$ is not connected. So, a point $p \in X$ is a *non-cut point* of X if $X - \{p\}$ is connected. An *arc* is a space homeomorphic to the unit interval $[0, 1]$.

The following theorem is well known:

Theorem 1. *A metric continuum is an arc if, and only if, it has exactly two non-cut points. (For detail, see Theorem 2-27 in [1] or Theorem 6.17 in [2], for example.)*

By following Section 28 in [3], we can generalize Theorem 1 to a Hausdorff arc in Definition 3 as follows:

Definition 2. A *cutting* of a continuum X is an ordered triple (p, U, V) where p is a cut point of X and U and V are disjoint non-empty open subsets of X whose union is $X - \{p\}$. A cut point p in a connected space X *separates a from b* if a cutting (p, U, V) exists with $a \in U$ and $b \in V$. The set consisting of a, b and all points p which separate a from b is denoted $E(a, b)$. The *separation order* on $E(a, b)$ is defined by: $p_1 \leq p_2$ if $p_1 = p_2$ or p_1 separates a from p_2. This is a partial order on $E(a, b)$. We write $p_1 < p_2$ if $p_1 \neq p_2$.

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