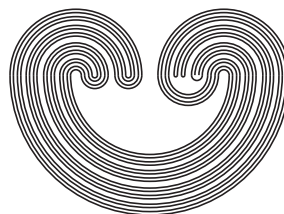


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## CONNECTEDNESS OF INVERSE LIMITS WITH SET-VALUED FUNCTIONS

by

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## CONNECTEDNESS OF INVERSE LIMITS WITH SET-VALUED FUNCTIONS

M. M. MARSH

**ABSTRACT.** We establish general results for determining connectedness of inverse limits on continua with set-valued bonding functions. These results generalize all theorems in the literature where connectedness of the inverse limit can be established by checking easily observable properties of the bonding functions. For inverse limits on  $[0, 1]$ , we note several useful special cases of our main theorem. The results provide answers to two questions of W. T. Ingram. We give a number of examples to illustrate the utility of the results.

### 1. INTRODUCTION

Unlike ordinary inverse sequences on continua with mappings for bonding functions, an inverse sequence on continua with set-valued bonding functions may not have a connected limit, even when the graphs of the bonding functions are continua. We establish sufficient conditions for connectedness of inverse limits on continua with upper semi-continuous set-valued functions. The conditions are simply-checked properties of the bonding functions. We also provide an additional result for connectedness of inverse limits on  $[0, 1]$  with set-valued bonding functions. The results provide answers to Problems 6.3 and 6.4 of Ingram in [2]. The results also generalize all theorems presently in the literature that give connectedness of an inverse limit with set-valued functions, where one only needs to observe that the bonding functions satisfy certain conditions. In the last section of the paper, we provide examples that illustrate the utility of the results.

A general introduction to results and questions related to connectedness of an inverse limit with set-valued functions can be found in Section

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