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by

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PAIRWISE DISJOINT REFINEMENTS OF COVERINGS OF ONE-DIMENSIONAL PEANO CONTINUA

KATSUYA EDA AND MARK MEILSTRUP

Abstract. For a given finite open cover $\mathcal{O}$ of a one-dimensional Peano continuum X, there exist refinements $\mathcal{O}_0$ and $\mathcal{O}_1$ of $\mathcal{O}$ consisting of connected open sets such that $\mathcal{O}_0 \cup \mathcal{O}_1$ covers X and both $\mathcal{O}_0$ and $\mathcal{O}_1$ are pairwise disjoint families.

In the proof of the so-called arc-reduced form theorem [4, Theorem 2.9] for one-dimensional Peano continua the second author used a fact, i.e. Theorem 1, without proof. Later, the first author gave a proof of the arc-reduced form theorem [2, Theorem 1.2] using brick partitions without using that fact. In this note we give a proof of this fact, namely:

Theorem 1. *For a given finite open cover $\mathcal{O}$ of a one-dimensional Peano continuum X, there exist refinements $\mathcal{O}_0$ and $\mathcal{O}_1$ of $\mathcal{O}$ consisting of connected open sets such that $\mathcal{O}_0 \cup \mathcal{O}_1$ covers X and both $\mathcal{O}_0$ and $\mathcal{O}_1$ are pairwise disjoint families.*

Since brick partitions are indispensable to our proof, we introduce brick partitions and related definitions.

Let X be a one-dimensional Peano continuum with its metric ρ. A metric space (X, ρ) is *uniformly locally connected* if, for every $\varepsilon > 0$, there exists $\delta > 0$ such that if $\rho(x, y) < \delta$ then x and y are contained in a connected open set of diameter less than ε.

A *brick partition* $\mathcal{P}$ of a space X is a collection of finitely many connected regular open sets such that the union $\bigcup_{P \in \mathcal{P}} P$ is dense in X and $int(\overline{P} \cup \overline{Q})$ is uniformly locally connected for each $P, Q \in \mathcal{P}$. Consequently each element of $\mathcal{P}$ is uniformly locally connected.

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