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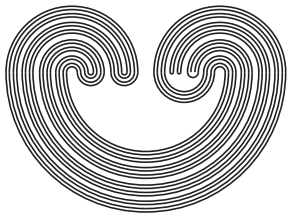

# Embeddings of 4–manifolds in $S^4 \tilde{\times} S^2$

by

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# EMBEDDINGS OF 4–MANIFOLDS IN $S^4 \tilde{\times} S^2$

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Abstract. In this paper, we discuss simplified broken achiral Lefschetz fibration embeddings (SBLFE) of simplified broken achiral Lefschetz fibrations (SBLF) of compact, connected, oriented, smooth 4–manifolds over $D^2$ into the trivial Lefschetz fibration of $\mathcal{D}E(1) \times D^2$ over $D^2$. Using this, we show that the double of an SBLF admits a smooth embedding in $S^4 \tilde{\times} S^2$. Also, we provide broken achiral Lefschetz fibration (BLF) embeddings of a huge collection of BLF's over $S^2$ into the trivial Lefschetz fibration of $S^4 \times S^2$ over $S^2$. Finally, given a closed oriented smooth 4−manifold $Y$, we provide BLF embeddings of a huge collection of BLF's over $S^2$ into the trivial Lefschetz fibration of $Y \times S^2$ over $S^2$.

## 1. Introduction

The embedding of a manifold into the Euclidean spaces as well as into a manifold with simple topology is one of the important problems in geometric topology. In this article, we are interested in co-dimension 2 embeddings of 4−manifolds using bordered Lefschetz fibrations.

In [11], bordered Lefschetz fibration embeddings of bordered achiral Lefschetz fibrations of compact, connected, oriented 4–manifolds over $D^2$ into the trivial Lefschetz fibration of $\mathcal{D}E(m) \times D^2$ over $D^2$ are discussed, where $\mathcal{D}E(m)$ is the 2–disc bundle over $S^2$ with the Euler number $m$. Using this, it is also shown that the double of a bordered achiral Lefschetz fibration of a 4–manifold $X$ admits a smooth embedding into $S^4 \times S^2$ as well as into $S^4 \tilde{\times} S^2$. Also in [11], bordered Lefschetz fibration embeddings of compact, connected, non-orientable 4–manifolds over $D^2$ into the

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