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TOPOLOGY PROCEEDINGS

Volume 63, 2024

Pages 125–148

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by

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Electronically published on September 16, 2023

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/

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E-Published on September 16, 2023

STRONGLY RIGID METRICS IN SPACES OF METRICS

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Abstract. A metric space is said to be strongly rigid if no positive distance is taken twice by the metric. In 1972, Janos proved that a separable metrizable space has a strongly rigid metric if and only if it is zero-dimensional. In this paper, we shall develop this result for the theory of spaces of metrics. For a strongly zero-dimensional metrizable space, we prove that the set of all strongly rigid metrics is dense in the space of metrics. Moreover, if the space is the union of countably many compact subspaces, then that set is comeager. As a consequence, we show that for a strongly zero-dimensional metrizable space, the set of all metrics possessing no nontrivial (bijective) self-isometry is comeager in the space of metrics.

1. Introduction

1.1. **Background.** Let X be a topological space, and S a subset of $[0, \infty)$ with $0 \in S$. We denote by $\mathrm{Met}(X; S)$ the set of all metrics on X taking values in S and generating the same topology of X. We also denote by $\mathcal{D}_X$ the supremum metric on $\mathrm{Met}(X; S)$; namely, $\mathcal{D}_X(d, e) = \sup_{x,y \in X} |d(x, y) - e(x, y)|$. We often write $\mathrm{Met}(X) = \mathrm{Met}(X; [0, \infty))$. Observe that $\mathcal{D}_X$ is a metric taking values in $[0, \infty]$. As is the case of ordinary metric spaces, we can introduce the topology on $\mathrm{Met}(X)$ generated by open balls. In what follows, we consider that $\mathrm{Met}(X)$ is equipped with this topology. In [7, 8, 9, 11], the author proved the denseness and

2020 *Mathematics Subject Classification.* Primary 54E35; Secondary 54E52, 51F99.

Key words and phrases. Strong rigidity, Rigidity, Space of metrics, Comeagerness, Baire space.