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HYPERSPACES OF ARCS CONTAINING A POINT

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Abstract. For a metric continuum X and for $p \in X$, we define the hyperspace $Arcs(p, X)$ as the set of all arcs in X that contain the point p, endowed with the Hausdorff metric. We show geometric models of $Arcs(p, X)$, in the case when X is the arc, the simple closed curve, the simple triod, the noose and the $\sin(\frac{1}{x})$-continuum. Moreover, we present a characterization of dendrites in terms of this hyperspace.

1. Introduction

A *continuum* is a compact, connected metric space with more than one point. A hyperspace is a collection of closed subsets having some common property. The study of the geometric models of hyperspaces is an interesting topic in continuum theory ([7] and [8, Chapter II]). Given a continuum X, the hyperspace of closed, nonempty subsets of X is denoted by 2^X and the hyperspace of subcontinua of X is denoted by $C(X)$. In 1978, Sam B. Nadler, Jr. [11, p. 601] suggested to study the hyperspace of arcs of a continuum X, defined and denoted by $\mathcal{A}(X) = \{A \subset X : A$ is an arc in $X\}$. In 1999, Adrián Soto [14] studied in his bachelor thesis the hyperspace of arcs and singletons of a continuum X, that is, the collection $\mathcal{M}(X) = \mathcal{A}(X) \cup \{\{x\} : x \in X\}$, and he obtained properties of $\mathcal{M}(X)$ when X is a dendroid. In 2002, Alejandro Illanes [6] gave a characterization of dendrites using the hyperspace of arcs and singletons.

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