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by

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TRANSCENDENTAL GROUPS

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Dedicated to Ralph Kopperman

Abstract. In this note we introduce the notion of a transcendental group, that is, a subgroup G of the topological group $\mathbb{C}$ of all complex numbers such that every element of G except 0 is a transcendental number. All such topological groups are separable metrizable torsion-free abelian groups. If $G \subset \mathbb{R}$, then G is also zero-dimensional and homeomorphic to a subspace of $\mathbb{N}^{\aleph_0}$, where $\mathbb{N}$ denotes the discrete space of natural numbers. It is shown that (i) each countably infinite transcendental group is a member of one of three classes, where each class has $\mathfrak{c}$ (the cardinality of the continuum) members – the first class consists of those isomorphic as a topological group to the discrete group $\mathbb{Z}$ of integers, the second class consists of those isomorphic as a topological group to $\mathbb{Z} \times \mathbb{Z}$, and the third class consists of those homeomorphic to the topological space $\mathbb{Q}$ of all rational numbers; (ii) for each cardinal number $\aleph$ with $\aleph_0 < \aleph \leq \mathfrak{c}$, there exist $2^{\aleph}$ transcendental groups of cardinality $\aleph$ such that no two of the transcendental groups are isomorphic as topological groups or even homeomorphic; (iii) there exist $\mathfrak{c}$ countably infinite transcendental groups each of which is homeomorphic to $\mathbb{Q}$ and algebraically isomorphic to a vector space over the field $\mathbb{A}$ of all algebraic numbers (and hence also over $\mathbb{Q}$) of countably infinite dimension; (iv) $\mathbb{R}$ has $2^{\mathfrak{c}}$ transcendental subgroups such that no two of the them are isomorphic as topological groups or even homeomorphic.

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