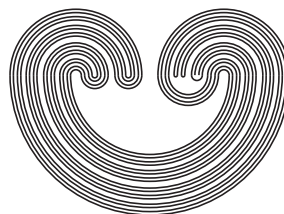


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TRANSCENDENTAL GROUPS

by

SIDNEY A. MORRIS

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TRANSCENDENTAL GROUPS

SIDNEY A. MORRIS

Dedicated to Ralph Kopperman

ABSTRACT. In this note we introduce the notion of a transcendental group, that is, a subgroup G of the topological group \mathbb{C} of all complex numbers such that every element of G except 0 is a transcendental number. All such topological groups are separable metrizable torsion-free abelian groups. If $G \subset \mathbb{R}$, then G is also zero-dimensional and homeomorphic to a subspace of \mathbb{N}^{\aleph_0} , where \mathbb{N} denotes the discrete space of natural numbers. It is shown that (i) each countably infinite transcendental group is a member of one of three classes, where each class has \mathfrak{c} (the cardinality of the continuum) members – the first class consists of those isomorphic as a topological group to the discrete group \mathbb{Z} of integers, the second class consists of those isomorphic as a topological group to $\mathbb{Z} \times \mathbb{Z}$, and the third class consists of those homeomorphic to the topological space \mathbb{Q} of all rational numbers; (ii) for each cardinal number \aleph with $\aleph_0 < \aleph \leq \mathfrak{c}$, there exist 2^\aleph transcendental groups of cardinality \aleph such that no two of the transcendental groups are isomorphic as topological groups or even homeomorphic; (iii) there exist \mathfrak{c} countably infinite transcendental groups each of which is homeomorphic to \mathbb{Q} and algebraically isomorphic to a vector space over the field \mathbb{A} of all algebraic numbers (and hence also over \mathbb{Q}) of countably infinite dimension; (iv) \mathbb{R} has $2^\mathfrak{c}$ transcendental subgroups such that no two of the them are isomorphic as topological groups or even homeomorphic.

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