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RECOGNITION AND RECONSTRUCTION OF SETS IN ℓ^2 VIA THEIR PROJECTIONS

STOYU T. BAROV

Abstract. Let $k \in \mathbb{N}$ and let $\mathcal{P}$ be a subset of all k-dimensional linear subspaces $\mathcal{G}_k$ of ℓ^2 with the natural topology. The subsets B and C of ℓ^2 are called $\mathcal{P}$-imitations of each other if $B + P = C + P$ for every $P \in \mathcal{P}$. In the case when $\mathcal{P}$ is somewhere dense G_δ-set in $\mathcal{G}_k$, we show that there are certain non-trivial sets in ℓ^2 such that each of them has only one $\mathcal{P}$-imitation, namely, itself. Consequently, every such set can be reconstructed as the intersection of the preimages of its projections under $\mathcal{P}$. In addition, we discuss some important properties of σ-compact sets that are of independent interest.

1. Introduction

The main purpose of the current note is to show that certain sets in ℓ^2 can be identified via their orthogonal projections and then, having somewhere dense G_δ-set $\mathcal{P}$ of directions, we can fully reconstruct each of them due to the fact that each set has no other $\mathcal{P}$-imitation than itself. In order to formulate our main results we need some definitions and notations. If $k \in \mathbb{N}$ then we let $\mathcal{G}_k$ denote the set of all k-dimensional linear subspaces of ℓ^2 with the natural topology; see Definition 2. If $B, C \subset \ell^2$ and $\mathcal{P} \subset \mathcal{G}_k$ then B and C are called *$\mathcal{P}$-imitations* of each other if $B + P = C + P$ for every $P \in \mathcal{P}$. If $\overline{B + P} = \overline{C + P}$ for every $P \in \mathcal{P}$ then B and C are called *weak $\mathcal{P}$-imitations* of each other. Further, if $A \subset \ell^2$ then the *geometric interior* A° of A is the interior of A relative to its closed affine hull. Finally, if L is a plane in ℓ^2 then $\psi_L \colon \ell^2 \to L^\perp$ denotes the orthogonal projection along L onto $L^\perp$. The starting point of the current research is the imitation theorem [4, Theorem 4]. It reads as follows.

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