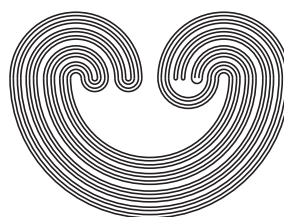


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by

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RECOGNITION AND RECONSTRUCTION OF SETS IN ℓ^2 VIA THEIR PROJECTIONS

STOYU T. BAROV

ABSTRACT. Let $k \in \mathbb{N}$ and let \mathcal{P} be a subset of all k -dimensional linear subspaces \mathcal{G}_k of ℓ^2 with the natural topology. The subsets B and C of ℓ^2 are called \mathcal{P} -imitations of each other if $B + P = C + P$ for every $P \in \mathcal{P}$. In the case when \mathcal{P} is somewhere dense G_δ -set in \mathcal{G}_k , we show that there are certain non-trivial sets in ℓ^2 such that each of them has only one \mathcal{P} -imitation, namely, itself. Consequently, every such set can be reconstructed as the intersection of the preimages of its projections under \mathcal{P} . In addition, we discuss some important properties of σ -compact sets that are of independent interest.

1. INTRODUCTION

The main purpose of the current note is to show that certain sets in ℓ^2 can be identified via their orthogonal projections and then, having somewhere dense G_δ -set \mathcal{P} of directions, we can fully reconstruct each of them due to the fact that each set has no other \mathcal{P} -imitation than itself. In order to formulate our main results we need some definitions and notations. If $k \in \mathbb{N}$ then we let \mathcal{G}_k denote the set of all k -dimensional linear subspaces of ℓ^2 with the natural topology; see Definition 2. If $B, C \subset \ell^2$ and $\mathcal{P} \subset \mathcal{G}_k$ then B and C are called \mathcal{P} -imitations of each other if $B + P = C + P$ for every $P \in \mathcal{P}$. If $\overline{B + P} = \overline{C + P}$ for every $P \in \mathcal{P}$ then B and C are called *weak \mathcal{P} -imitations* of each other. Further, if $A \subset \ell^2$ then the *geometric interior* A° of A is the interior of A relative to its closed affine hull. Finally, if L is a plane in ℓ^2 then $\psi_L: \ell^2 \rightarrow L^\perp$ denotes the orthogonal projection along L onto L^\perp . The starting point of the current research is the imitation theorem [4, Theorem 4]. It reads as follows.

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Key words and phrases. separable Hilbert space, orthogonal projection, set with empty geometric interior, reconstruction of a set, dense G_δ -set.

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