

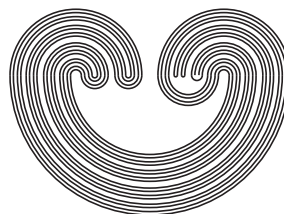
---

# TOPOLOGY PROCEEDINGS

Volume 63, 2024

Pages 191–209

---



## AN APPLICATION OF DESCRIPTIVE SET THEORY TO COMPLEX ANALYSIS

by

CHRISTOPHER J. CARUVANA AND ROBERT R. KALLMAN

Electronically published on February 4, 2024

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.nipissingu.ca/tp/subscriptioninfo.html> for information.

---

### Topology Proceedings

**Web:** <http://topology.nipissingu.ca/tp/>

**Mail:** Topology Proceedings  
Department of Mathematics & Statistics  
Auburn University, Alabama 36849, USA

**E-mail:** [topolog@auburn.edu](mailto:topolog@auburn.edu)

**ISSN:** (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

## AN APPLICATION OF DESCRIPTIVE SET THEORY TO COMPLEX ANALYSIS

CHRISTOPHER J. CARUVANA AND ROBERT R. KALLMAN

**ABSTRACT.** The purpose of this paper is to prove a new general result about rings of complex analytic functions. Let  $\Omega$  be an arbitrary nonempty open subset of the complex plane  $\mathbb{C}$ ,  $\mathcal{A}(\Omega)$  be the set of holomorphic functions on  $\Omega$  viewed as a Polish ring (not a Polish algebra over  $\mathbb{C}$ ) in the usual topology of uniform convergence on compact subsets of  $\Omega$ , let  $R$  be a Polish ring and let  $\varphi : R \rightarrow \mathcal{A}(\Omega)$  be an abstract algebraic isomorphism. The main goal of this paper is to prove Theorem 5.7 that  $\varphi$  is a topological isomorphism. This result may be viewed as a strengthening of the assertion that there is only one Polish ring topology on  $\mathcal{A}(\Omega)$ . A special result of Bers is an easy corollary. Two additional items supplement these results, viz., that  $B(\mathbb{D})$ , the abstract ring of bounded analytic functions on the unit disk, cannot be made into a Polish ring (Theorem 7.5) and that  $\mathcal{M}(\Omega)$ , the abstract field of meromorphic functions on  $\Omega$ , cannot be made into a Polish field (Theorem 8.1).

### 1. INTRODUCTION

As discussed in the abstract, the purpose of this paper is to prove the new general result about rings of complex analytic functions given in Theorem 5.7. The by now well known very general approach to the proof of theorems like Theorem 5.7 is to show that  $\varphi$  is measurable with respect to the sets with the Baire property. However, in almost every proof of such

---

2020 *Mathematics Subject Classification.* 30H50, 54H05, 54H13.

*Key words and phrases.* Descriptive set theory, Polish rings, Functions of a complex variable.

©2024 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.nipissingu.ca/tp/subscriptioninfo.html> for information.