http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS

Volume 63, 2024

Pages 211–225

# A topology on homotopy set induced by pairings

by

Nobuyuki Oda and Toshihiro Yamaguchi

Electronically published on February 14, 2024



**Topology Proceedings**

| | |
|---|---|
| **Web:** | http://topology.nipissingu.ca/tp/ |
| **Mail:** | Topology Proceedings<br>Department of Mathematics & Statistics<br>Auburn University, Alabama 36849, USA |
| **E-mail:** | topolog@auburn.edu |
| **ISSN:** | (Online) 2331-1290, (Print) 0146-4124 |

http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 63 (2024)
Pages 211-225

E-Published on February 14, 2024

# A TOPOLOGY ON HOMOTOPY SET INDUCED BY PAIRINGS

NOBUYUKI ODA AND TOSHIHIRO YAMAGUCHI

Abstract. Pairings of based topological spaces define perpendicular relations among base point preserving continuous maps. A topology on a base point preserving homotopy set is defined by making use of the perpendicular relation. The topological space thus obtained is an Alexandroff space. Some conditions are obtained for the induced function to be continuous. The dual results are also studied on the basis of copairings. Rational examples due to Arkowitz and Lupton whose homotopy classes of selfmaps are finite sets are studied in detail.

## 1. Introduction

All spaces and all maps in this paper are assumed to be topological spaces with base point $*$ and base point preserving continuous maps, respectively. We denote the constant map by $* : X \to Y$. We consider base point preserving homotopies and for two pointed spaces $X$ and $Y$, the symbol $[X, Y]$ denotes the base point preserving homotopy classes of maps $X \to Y$. Also a map $f : X \to Y$ is tacitly identified with its homotopy class $[f] \in [X, Y]$; we use the same symbol $\alpha$, $\beta$, $\gamma$, ... and $f$, $g$, $h$, ... for maps and homotopy classes. If $\alpha : A \to B$ is a map or a homotopy class, we write $A = s(\alpha)$ (the source of $\alpha$) and $B = t(\alpha)$ (the target of $\alpha$).

2020 *Mathematics Subject Classification.* 54C50, 55P30, 55P62, 55P65, 55Q05.
*Key words and phrases.* homotopy set, pairing, copairing, Alexandroff space.