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STAR LINDELÖFNESS OF PRODUCTS OF SUBSPACES OF ORDINALS

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Abstract. For an infinite cardinal κ, a topological space X is called κ-compact if every $F \subseteq X$ with $|F| \geq \kappa$ has an accumulation point. A space X is said to be star countable (respectively, star Lindelöf) if for every open cover $\mathcal{U}$ of X, there exists a countable subset (respectively, a Lindelöf subspace) F of X such that $\mathrm{St}(F,\mathcal{U}) = X$. In this paper, we give a characterization when the product $\prod_{i \leq n} A_i$ is κ-compact, where $\kappa > \omega$ is regular, n is a natural number, and each A_i is a subspace of an ordinal $\lambda_i + 1$. By using this result, we show that such a product $\prod_{i \leq n} A_i$ is star countable if and only if it is star Lindelöf.

1. Introduction

All spaces considered are Hausdorff topological spaces. As usual, an ordinal is equal to the set of smaller ordinals. A subset of an ordinal λ is assumed to have the relative topology induced by the order topology on λ. The symbol ω denotes the first infinite ordinal, ω_1 denotes the first uncountable ordinal, and $\mathbb{N}$ denotes the set of all positive integers.

Let X be a space and $\mathcal{U}$ a collection of subsets of X. For $A \subseteq X$, let $\mathrm{St}(A,\mathcal{U}) = \bigcup\{U \in \mathcal{U} : U \cap A \neq \emptyset\}$. As usual, we write $\mathrm{St}(x,\mathcal{U})$ instead of $\mathrm{St}(\{x\},\mathcal{U})$. Recall that a space X is *star countable* (respectively, *star Lindelöf*) if for each open cover $\mathcal{U}$ of X, there is a countable set $F \subseteq X$ (respectively, a Lindelöf subspace F) such that $\mathrm{St}(F,\mathcal{U}) = X$.

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