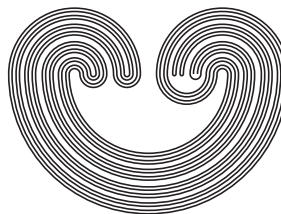


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## STAR LINDELÖFNESS OF PRODUCTS OF SUBSPACES OF ORDINALS

by

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## STAR LINDELÖFNESS OF PRODUCTS OF SUBSPACES OF ORDINALS

LEI MOU, YANHUI HUANG, AND YANQIN XU

**ABSTRACT.** For an infinite cardinal  $\kappa$ , a topological space  $X$  is called  $\kappa$ -compact if every  $F \subseteq X$  with  $|F| \geq \kappa$  has an accumulation point. A space  $X$  is said to be star countable (respectively, star Lindelöf) if for every open cover  $\mathcal{U}$  of  $X$ , there exists a countable subset (respectively, a Lindelöf subspace)  $F$  of  $X$  such that  $\text{St}(F, \mathcal{U}) = X$ . In this paper, we give a characterization when the product  $\prod_{i \leq n} A_i$  is  $\kappa$ -compact, where  $\kappa > \omega$  is regular,  $n$  is a natural number, and each  $A_i$  is a subspace of an ordinal  $\lambda_i + 1$ . By using this result, we show that such a product  $\prod_{i \leq n} A_i$  is star countable if and only if it is star Lindelöf.

### 1. INTRODUCTION

All spaces considered are Hausdorff topological spaces. As usual, an ordinal is equal to the set of smaller ordinals. A subset of an ordinal  $\lambda$  is assumed to have the relative topology induced by the order topology on  $\lambda$ . The symbol  $\omega$  denotes the first infinite ordinal,  $\omega_1$  denotes the first uncountable ordinal, and  $\mathbb{N}$  denotes the set of all positive integers.

Let  $X$  be a space and  $\mathcal{U}$  a collection of subsets of  $X$ . For  $A \subseteq X$ , let  $\text{St}(A, \mathcal{U}) = \bigcup \{U \in \mathcal{U} : U \cap A \neq \emptyset\}$ . As usual, we write  $\text{St}(x, \mathcal{U})$  instead of  $\text{St}(\{x\}, \mathcal{U})$ . Recall that a space  $X$  is *star countable* (respectively, *star Lindelöf*) if for each open cover  $\mathcal{U}$  of  $X$ , there is a countable set  $F \subseteq X$  (respectively, a Lindelöf subspace  $F$ ) such that  $\text{St}(F, \mathcal{U}) = X$ .

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*Key words and phrases.* extent,  $\kappa$ -compact, ordinal, product, star countable, star Lindelöf.

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