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by

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ARCWISE CONNECTEDNESS OF A HYPERSPACE OF NON-CONNECTED SUBSPACES OF A CONTINUUM

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Abstract. For a metric continuum X, let $C_n(X)$ be the hyperspace of nonempty closed subsets of X with at most n components. Answering a question by J. Camargo and S. Macías, in this paper we prove that if $n \geq 2$ and X is a hereditarily decomposable continuum not containing terminal subcontinua, then $C_n(X) \setminus C_1(X)$ is arcwise connected.

1. Introduction

A *continuum* is a nondegenerate, compact, connected, metric space. A *mapping* is a continuous function. Given a continuum X, a subcontinuum of X is a nonempty, closed, connected subset of X, so one-point subsets of X are subcontinua of X. For $n \in \mathbb{N}$, we consider the following hyperspaces of X.

$$2^X = \{A \subset X : A \text{ is closed and nonempty}\},$$
$$C_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ components}\}, \text{ and}$$
$$F_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ points}\}.$$

These hyperspaces are considered with the Hausdorff metric [3, Theorem 2.2]. As usual, the hyperspace $C_1(X)$ is denoted by $C(X)$.

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