Fields Institute Workshop on Topological Methods in Algebra, Analysis, and Dynamical Systems

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Abstracts

Department of Computer Science and Mathematics
Nipissing University
North Bay, Ontario, Canada

An introduction to Weingarten Calculus

Benoit Collins

University of Ottawa

In these two lectures I will describe a general method for computing integrals on compact matrix groups with respect to their Haar measure. The integration formulas involve combinatorial sums, and they were first described by physicists including Hooft, Weingarten, Itzykson and Zuber. I will survey some recent mathematical developments, and in particular, applications to random matrix theory, operator algebra and probability theory.

Complex Dynamics and Complex Topology

Robert L. Devaney

Boston University

In these talks we will describe some of the numerous intersections of complex dynamics with topology. The Julia sets for complex rational maps are often extremely complicated topological spaces but nevertheless are spaces that topologists often understand very well. The problem that arises is how to understand the dynamics on these interesting sets. We will concentrate on singularly perturbed rational maps, i.e., maps of the form $z^n + \lambda/z^d$ where λ is a complex parameter. For these maps, many of the Julia sets are Sierpinski curves and hence are well understood from the topological point of view. Yet the dynamics on these sets vary wildly as the parameters change. We will describe a partial classification of the dynamical behavior on these sets and explore more complicated singular perturbations such as those generated by $z^2 + c + \lambda/z^d$, i.e., singular perturbations of the quadratic polynomials that arise in the Mandelbrot set. Here the topology is less well understood and the dynamics are even more complicated.

Topological centres, admissible algebra and semigroup compactifications

Stefano Ferri

Universidad de los Andes (Bogota)

Matthias Neufang

Carleton University

Given a Banach algebra B there are two natural products on its second dual B'' called the first a second Arens product. A natural way of measuring its Arens irregularity is to consider the sets of elements in the bidual for which left (resp. right) multiplication with respect to both Arens products is the same, called the left (resp. right) topological centre. We have an analogous notion for certain quotient algebras of the bidual, and in the context of semigroup compactifications. In the first lectures recent progress which have been made in determining the topological centre for various algebras arising in abstract harmonic analysis will be discussed. In the last lecture we shall concentrate on semigroup compactifications and discuss their relation to unitary and reflexive representability of topological groups. We shall particularly focus on the following:

- the topological centre of the group algebra, the measure algebra, the algebra $LUC(G)^*$ and weighted convolution algebras over locally compact groups, as well as the LUC-compactification G^{LUC} of G [mostly results of Matthias Neufang];
- the $LUC(G)^*$ and G^{LUC} for unbounded, separable (not necessarily locally compact) groups [joint work with Matthias Neufang];
- the number of elements in $LUC(G)^*$ resp. G^{LUC} needed to "determine" the topological centre [closely related to recent work by G. Dales, A. T.-M. Lau and D. Strauss];
- the size of the quotient space $LUC(G)/WAP(G)$ and applications to extreme non Arens regularity of the group algebra [Matthias Neufang and C.K. Fong];
- applications to uniqueness of invariant means on general topological groups, and a characterization of equi left uniform continuity in the locally compact case;
- semigroup compactifications and unitarily and reflexive representability [joint work with Jorge Galindo Pastor];
- reflexive representability and stability [joint work with Itai Ben Yaakov and Alexander Berenstein].

Part of the course will follow the structure (and use part of the slides) of early talks given by Matthias Neufang.

Julia sets of elliptic functions: a compendium of topological types

Jane Hawkins

University of North Carolina at Chapel Hill

We give an overview of the theory behind iteration of elliptic functions, that is, doubly periodic meromorphic maps. We discuss the dependence of the dynamics of the Weierstrass elliptic P function and other elliptic maps on not only the period lattice shape, but on the lattice side length as well. We show that many topological types of

Julia sets occur; in particular one type that cannot be seen among rational maps (Fatou toral bands), along with Sierpinski carpets, and parametrized families of maps with Julia set the whole sphere. We will review results from the literature as well as mention new work and an open question on the occurrence of Cantor Julia sets.

Homotopy groups of some compact metric spaces which are related to the Hawaiian earring

Kazuhiro Kawamura

Institute of Mathematics, University of Tsukuba, Japan

Co-author: Katsuya Eda

Waseda University, Japan

The behavior of singular (co)homology groups and homotopy groups of compact metric spaces is sometimes counter-intuitive. One of such examples is provided by the failure of Seifert-van Kampen Theorem for the one-point union $CH \vee CH$ of two copies of the cone CH over the Hawaiian earring H. We discuss singular homology and homotopy groups of continua which are related to the Hawaiian earring. In particular we point out that the space $CH \vee CH$ is aspherical in the sense that each homotopy group of dimension at least two is trivial. This is a joint work with Katsuya Eda, Waseda University, Japan.

Inverse limits, economics and backward dynamics

Judy Kennedy

Lamar University and University of Delaware

Co-authors: Brian Raines, David R.Stockman, and Jim Yorke

Some economic models such as the cash-in-advance model of money have the property that the dynamics are ill-defined going forward in time, but well-defined going backward in time. We apply the theory of inverse limits to characterize topologically possible solutions to a dynamic economic model with this property. We show that such techniques are particularly well-suited for analyzing the dynamics going forward in time even though the dynamics are ill-defined in this direction. We analyze the inverse limit of the cash-in-advance model of money and illustrate how information about the inverse limit is useful for detecting or ruling out complicated dynamics.

We have also been able to put an appropriate measure on the inverse limit, which makes it possible to integrate continuous functions over the inverse limit space. It is then possible to compute expected utility for a given cash-in-advance model, and we are able to rank models according to their expected utility.

Applications of rings of continuous functions in the theory of locally precompact groups

Gabor Lukács

University of Manitoba

Talk 1.

Let X be a topological space, and consider the set $C(X)$ of continuous real-valued maps on X. By equipping $C(X)$ with *pointwise* operations, one obtains a commutative ring structure. It turns out that the sets of points where continuous functions vanish carry a lot of interesting information, just like in algebraic geometry. The difference is, however, that in algebraic geometry, the zero set of a proper ideal is never empty (over an algebraically closed field), whereas if X is not (pseudo)compact, then $C(X)$ contains many proper ideals whose "classical" zero set is empty. In this talk, the classical theory of rings of continuous functions is surveyed (cf. [5]), with emphasis on the following two questions: What can one learn about the space X by studying properties of the ring $C(X)$? What are the "interesting" ideals of $C(X)$?

Talk 2. (Derived from joint work with W. W. Comfort)

A subset B of a (Hausdorff) topological group G is said to be *precompact* if for every neighborhood U of the identity in G, there is a finite subset $F \subseteq G$ such that $B \subseteq (FU) \cap (UF)$. An interesting subclass of the class of precompact groups was identified and studied by Comfort and Ross, who showed *inter alia* that a topological group G is pseudocompact if and only if it is precompact and G_δ-dense in its completion $\widetilde{G}$ (cf. [3]). Since then, precompact groups have been a focus of interest (cf. [7], [1]).

A group G is *locally precompact* if it contains a precompact neighborhood of the identity. The completion of a locally precompact group is locally compact (cf. [9]), and thus such groups are precisely the subgroups of locally compact groups. Comfort and Trigos-Arrieta extended the Comfort-Ross criterion, and proved that a locally precompact group G is locally pseudocompact if and only if it is G_δ-dense in $\widetilde{G}$ (cf. [4]). Locally pseudocompact groups were also studied by Sanchis (cf. [8]).

In this talk, we present characterizations of the following properties within the class of locally precompact groups:

(a) local realcompactness;
(b) topological completeness;
(c) realcompactness;
(d) hereditary realcompactness.

References

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Any counterexample to the Makienko conjecture is an indecomposable continuum

John C. Mayer

University of Alabama at Birmingham

Co-authors: Clinton Curry (UAB), Jonathan Meddaugh (Tulane), James T. Rogers, Jr. (Tulane)

The residual Julia set of a rational function is defined as its Julia set minus the boundaries of its Fatou components. It follows from the Baire Category Theorem and properties of Julia sets that, when a component of the Fatou set is fully invariant under some power of the map, the residual Julia set is empty. Based on Sullivan's dictionary, Peter M. Makienko conjectured that the converse is true: when the residual Julia set of a rational map is empty, there is a Fatou component which is fully invariant under a power of the map. Until now, this has been confirmed only for Julia sets which are not connected and for Julia sets which are locally connected. We prove that any counterexample to Makienko's conjecture is an indecomposable continuum. It is not known if indecomposable Julia sets exist.

Geometric methods for plane continua I

Lex Oversteegen

University of Alabama at Birmingham

Conformal maps have been a power full tool for studying plane continua. Given a non-separating continuum X in the sphere $\mathbb{S}$, there exists a conformal map $\varphi\colon \mathbb{D}^\infty \to \mathbb{S}\setminus X = U$, where $\mathbb{D}^\infty$ is the complement of the closed unit disk in the sphere. The map φ provides a way to impose a coordinate system on U: one can consider the images of round circles (called *level curves*) and the images of radial line segments with argument $2\pi\alpha$ (called the *external ray* R_α). However, this approach is not satisfactory under metric perturbations of X (for example an isotopy of X). In this talk we will provide a new coordinate system which will rely only on the metric and, hence, is easy to use when X is continuously moved. Our goal will be to provide a description of metrically defined external rays and level curves and describe a natural way to partition U into disjoint closed sets.

Geometric methods for plane continua II. Applications

Lex Oversteegen

University of Alabama at Birmingham

In this talk we will apply the results of the first talk to give a characterization of an accessible point in a planar continuum. Then we show that this property is preserved under isotopy of X and use it show that any isotopy of a plane continuum extends to an isotopy of the entire plane. This extends the fact that a holomorphic motion of a subset of the plane always extends to the entire plane

Banach representations and enveloping semigroups

Vladimir Uspenskiy

Ohio University

By a *topological dynamical system* (TDS) (G, X) we mean a topological group G which continuously acts on a compact space X. With every Banach space V one can associate the TDS (G, X), where G is the topological group of isometries of V, and X is the unit ball of the dual space V^* equipped with the w^*-topology. Every TDS can be embedded in a TDS arising in this way from a Banach space V, and such embeddings can be used to better understand topological groups and dynamical systems.

The enveloping semigroup of a TDS (G, X) is the closure of the set of all g-translations, $g \in G$, in the compact space X^X. A topological group G is *extremely amenable* if every compact G-space has a fixed point.

A selection of topics that we are going to discuss:

- weakly almost periodic functions, strongly exposed points, and representations on reflexive spaces;
- the Radon-Nikodym property of convex sets and representations on Asplund spaces;
- the group $H_+[0, 1]$ of all orientation-preserving self-homeomorphisms of $[0, 1]$ does not admit non-trivial representations on Banach spaces with a separable dual;
- the Urysohn space and its group of isometries;
- the Roelcke compactification of a topological group;
- Glasner's problem: are Abelian minimally almost periodic groups extremely amenable?