Applications of rings of continuous functions in the theory of locally precompact groups

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ABSTRACT

Talk 1. Let X be a topological space, and consider the set $C(X)$ of continuous real-valued maps on X. By equipping $C(X)$ with *pointwise* operations, one obtains a commutative ring structure. It turns out that the sets of points where continuous functions vanish carry a lot of interesting information, just like in algebraic geometry. The difference is, however, that in algebraic geometry, the zero set of a proper ideal is never empty (over an algebraically closed field), whereas if X is not (pseudo)compact, then $C(X)$ contains many proper ideals whose "classical" zero set is empty. In this talk, the classical theory of rings of continuous functions is surveyed (cf. [5]), with emphasis on the following two questions: What can one learn about the space X by studying properties of the ring $C(X)$? What are the "interesting" ideals of $C(X)$?

Talk 2. (Derived from joint work with W. W. Comfort) A subset B of a (Hausdorff) topological group G is said to be *precompact* if for every neighborhood U of the identity in G, there is a finite subset $F \subseteq G$ such that $B \subseteq (FU) \cap (UF)$. An interesting subclass of the class of precompact groups was identified and studied by Comfort and Ross, who showed *inter alia* that a topological group G is pseudocompact if and only if it is precompact and G_δ-dense in its completion $\widetilde{G}$ (cf. [3]). Since then, precompact groups have been a focus of interest (cf. [7], [1]).

A group G is *locally precompact* if it contains a precompact neighborhood of the identity. The completion of a locally precompact group is locally compact (cf. [9]), and thus such groups are precisely the subgroups of locally compact groups. Comfort and Trigos-Arrieta extended the Comfort-Ross criterion, and proved that a locally precompact group G is locally pseudocompact if and only if it is G_δ-dense in $\widetilde{G}$ (cf. [4]). Locally pseudocompact groups were also studied by Sanchis (cf. [8]).

In this talk, we present characterizations of the following properties within the class of locally precompact groups:

(a) local realcompactness;
(b) topological completeness;
(c) realcompactness;
(d) hereditary realcompactness.

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