Monotone Decompositions for Julia Sets

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A *monotone decomposition* of a topological space X is a partition Q of X into disjoint connected subsets. The corresponding quotient map $X \to X/Q$ is a *monotone map* (a map whose point inverses are connected). A successful and venerable strategy for studying a class of spaces is to study their monotone decompositions. This strategy has been employed for quite some time, most famously by Kuratowski in his work on the internal structure of irreducible continua.

We are interested in studying the monotone decompositions of (connected) Julia sets of rational maps. If R is a rational map on the Riemann sphere, its *Julia set* $J(R)$ is the set upon which R exhibits sensitive dependence on initial conditions. (It can also be characterized as the closure of the set of repelling periodic points of R, or as the complement of the domain of normality of the iterates of R.) The implicit definition of $J(R)$ potentially introduces difficulty in applying the strategy above, and in fact the general topological properties of Julia sets are still mysterious. We will discuss a few different aspects of the problem, including the following questions:

(1) What Julia sets are finitely irreducible continua (i.e., contain a finite subset A such that no proper subcontinuum contains A)?

(2) When does a Julia set have a monotone decomposition to a locally connected continuum which respects the dynamics? (We have an answer for polynomials.)

(3) What sort of decomposition is appropriate for arbitrary rational functions?