An Extended Fatou-Shishikura inequality and wandering branch continua for polynomials

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Let $P$ be a polynomial of degree $d$ with Julia set $J_P$. Let $\tilde{N}$ be the number of non-repelling cycles of $P$. By the famous Fatou-Shishikura inequality, $\tilde{N} \leq d - 1$. The goal of the paper is to improve this bound. The new count includes wandering collections of eventual branch continua, i.e., collections of continua or points $Q_i \subset J_P$ all of whose images are pairwise disjoint and, if taken for sufficiently big powers of $P$, contain the limit sets of $\text{eval}(Q_i) \geq 3$ external rays. Also, we relate individual cycles, which are either non-repelling or repelling with no periodic rays landing, to individual critical points that are recurrent in a weak sense.

A weak version of the inequality reads

$$\tilde{N} + N_{irr} + \chi + \sum_i (\text{eval}(Q_i) - 2) \leq d - 1$$

where $N_{irr}$ counts repelling cycles with no periodic rays landing at points in the cycle, $\{Q_i\}$ form a wandering collection $\mathcal{B}_C$ of eventual branch continua, $\chi = 1$ if $\mathcal{B}_C$ is non-empty, and $\chi = 0$ otherwise.

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