An Extended Fatou-Shishikura inequality and wandering branch continua for polynomials

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Let P be a polynomial of degree d with Julia set J_P. Let $\widetilde{N}$ be the number of non-repelling cycles of P. By the famous Fatou-Shishikura inequality, $\widetilde{N} \leq d - 1$. The goal of the paper is to improve this bound. The new count includes *wandering collections of eventual branch continua*, i.e., collections of continua or points $Q_i \subset J_P$ *all* of whose images are pairwise disjoint and, if taken for sufficiently big powers of P, contain the limit sets of $\mathrm{eval}(Q_i) \geq 3$ external rays. Also, we relate individual cycles, which are either non-repelling or repelling with no periodic rays landing, to individual critical points that are recurrent in a weak sense.

A weak version of the inequality reads

$$\widetilde{N} + N_{irr} + \chi + \sum_i (\mathrm{eval}(Q_i) - 2) \leq d - 1$$

where N_{irr} counts repelling cycles with no periodic rays landing at points in the cycle, $\{Q_i\}$ form a wandering collection $\mathcal{B}_{\mathbb{C}}$ of eventual branch continua, $\chi = 1$ if $\mathcal{B}_{\mathbb{C}}$ is non-empty, and $\chi = 0$ otherwise.

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