Erdős type spaces of higher descriptive complexity

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Abstract: Let $(E_n)_{n\in\omega}$ be a sequence of zero-dimensional subsets of the reals, $\mathbb{R}$. The Erdős type space $\mathcal{E}$ corresponding to this sequence is defined by $\mathcal{E} = \{x \in \ell 2 : x_n \in E_n, n \in \omega\}$. This concept generalizes Erdős space, with E_n equal to the rationals for each n and complete Erdős space, with $E_n = \{0\} \cup \{1/m : m \in \mathbb{N}\}$ for each n; Erdős' famous examples in Dimension Theory. If all sets E_n are G_δ and the space $\mathcal{E}$ is not zero-dimensional, then $\mathcal{E}$ is known to be homeomorphic to complete Erdős space and if all sets E_n are $F_{\sigma\delta}$, then under a mild additional condition $\mathcal{E}$ is known to be homeomorphic to Erdős space. In terms of complexity Erdős and complete Erdős space represent the two simplest cases. In this talk we introduce an entire family of Erdős type spaces to represent cases for higher Borel complexities. We also introduce coanalytic Erdős space, which corresponds to Erdős type spaces constructed with coanalytic sets E_n, but also, we conjecture, to certain homeomorphism groups.

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