

Cell structures

Edward D. Tymchatyn (University of Saskatchewan)
tymchat@math.usask.ca

Abstract: A graph consists of a discrete set of vertices together with a reflexive and symmetric relation on the set of vertices which describes the edges. A cell structure is an inverse system of graphs with bonding maps which satisfy mild continuity and convergence conditions. We show that cell structures suffice to obtain a class of Tychonov spaces which includes compact Hausdorff spaces and complete metric spaces as perfect images of 0-dimensional spaces. Continuous maps between spaces are obtained from cell maps between cell structures. We regard cell structures as providing a kind of bridge between discrete and continuous mathematics. Historically, spaces and maps were obtained using inverse systems of polyhedra and later by Mardesic's resolutions and approximate resolutions. However, in many important cases it is impossible to obtain spaces or their maps with those methods using commutative diagrams. One is forced to use instead diagrams that are only approximately commutative. This makes use of inverse systems and resolutions difficult. Because we use only 1-dimensional data from covers of spaces commutative diagrams suffice for cell structures and cell maps. We expect this work to be of real interest to computer scientists and to people doing topological data analysis.