

Hyperspaces of Euclidean spaces in the Gromov-Hausdorff metric

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Abstract: The Gromov-Hausdorff distance d_{GH} is a useful tool for studying topological properties of families of metric spaces. For two compact metric spaces X and Y the number $d_{GH}(X, Y)$ is defined to be the infimum of all Hausdorff distances $d_H(i(X), j(Y))$ for all metric spaces M and all isometric embeddings $i : X \rightarrow M$ and $j : Y \rightarrow M$.

Clearly, the Gromov-Hausdorff distance between isometric spaces is zero; it is a metric on the family GH of isometry classes of compact metric spaces. The metric space $(GH; d_{GH})$ is called the Gromov-Hausdorff hyperspace. We mainly are interested in the subspace $GH(\mathbb{R}^n)$ of GH consisting of the classes $[E] \in GH$ whose representative E is a metric subspace of the Euclidean space \mathbb{R}^n , $n \geq 1$. $GH(\mathbb{R}^n)$ is called the Gromov-Hausdorff hyperspace of \mathbb{R}^n . One of the main results in this talk asserts that $GH(\mathbb{R}^n)$ is homeomorphic to the orbit space $2^{\mathbb{R}^n}/E(n)$, where $2^{\mathbb{R}^n}$ is the hyperspace of all nonempty compact subsets of \mathbb{R}^n endowed with the Hausdorff metric and $E(n)$ is the isometry group of \mathbb{R}^n . This is applied to prove that $GH(\mathbb{R}^n)$ is homeomorphic to the Hilbert cube with a removed point.