

## On Counterexamples to the Conjecture of Wood

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*Abstract:* The following conjecture [6], in the area of Functional Analysis, was formulated by G.V. Wood in 1982:

Suppose  $L$  is a locally compact Hausdorff space such that the Banach space of all continuous scalar-valued functions vanishing at infinity, supplied with the supremum norm, is almost transitive. Then  $L$  consists of a single point.

In 1997 Greim and Rajagopalan [1] confirmed Wood's Conjecture in the real case. In 2005 Kawamura [3] and Rambla [4], independently, constructed a metric counterexample  $L$  in the complex case, with  $L$  homeomorphic to the pseudo-arc with a point removed. Their result, in fact, turns out to be closely related to the property of projective homogeneity that characterizes the pseudo-arc among chainable continua, as shown by Irwin and Solecki in 2006 [2]. By the work of Greim and Rajagopalan [1] the metric counterexample gives rise to a nonmetric one, by an ultrafilter construction (see also [5]). We exhibit a new nonmetric counterexample and discuss its topological properties. Up to our knowledge, this is only the third known counterexample. We also show that, contrary to what was expected, removing an arbitrary point from R.H. Bing's pseudo-circle does not give a metric counterexample. This is joint work with Michel Smith.

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[5] F. C. Sánchez, *Transitivity of  $M$ -spaces and Wood's conjecture*, Math. Proc. Cambridge Phil. Soc. 124 (1998)

[6] G. V. Wood, *Maximal symmetry in Banach spaces*, Proc. Roy. Irish Acad. Sect. A 82 (1982)