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## Homogeneity and Actions of Topological Groups

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*Abstract*: A space $X$ is homogeneous if for every $x, y \in X$ there is a homeomorphism $h$ of $X$ such that $h(x) = y$. If $X$ is a homogeneous space then the group $\mathbf{H}(X)$ of its homeomorphisms naturally acts on $X$. If $\mathbf{H}(X)$ is endowed with the discrete topology then this action is continuous and transitive. But the discrete topology is not interesting.

Question 1. What topological groups acts continuously and transitively on a homogeneous space $X$?

Let $X$ be a coset space of a topological group $G$. Then the natural action of $G$ on $X$ is continuous, transitive and, moreover, it is open, i.e. that for any point $x \in X$ and any nbd $O$ of $e$ in $G$

$$x \in \mathrm{Int}(Ox) \text{ where } Ox = \{y \in X : y = h(x), h \in O\}.$$

Question 2. Let $X$ be a coset space. Can $X$ be a coset space of a topological group from some class of groups ((metrizable) compact, Polish, $\omega$-narrow, $\omega$-balanced, etc.)

From results of R. Arens and E. Effros, G. Ungar deduced that any homogeneous compact metrizable space is a coset space of a Polish group. From results of R. Arens and L. Kristensen it follows that metrizable compact space is a coset space of a compact metrizable group iff it is metrically homogeneous. N. Okromeshko showed that the class of coset spaces of compact groups coincides with the class of isometrically homogeneous compacta. J. van Mill showed that separable metrizable and Polish SLH spaces are coset spaces of separable metrizable and Polish groups respectively.

The preliminary investigation of Questions 1 and 2 will be presented.