14th Annual Workshop on Topology and Dynamical Systems
Nipissing University, May 15–19, 2017
http://topology.nipissingu.ca/workshop2017/

Critical Portraits of Complex Polynomials

John C. Mayer (University of Alabama, Birmingham)
jcmayer@uab.edu

Abstract: A complex polynomial P of degree d for which all orbits of critical points converge to attracting periodic orbits is said to exhibit hyperbolic dynamics. We consider those for which no critical point is attracted to the attracting fixed point at infinity. For such polynomials, it is well-known that the Julia set $J(P)$ is connected and locally connected. To such a Julia set, there corresponds a *lamination*, a collection of non-crossing chords in the unit disk whose quotient space formed by shrinking the chords to points is dynamically equivalent to $J(P)$.

We propose a high level view of the parameter space of such hyperbolic polynomials through the concept of critical portraits in the context of laminations of the unit disk. For degree d, a unit disk with a maximal number of non-crossing chords of critical length (each length k/d for some k) that can only meet at endpoints is called a *critical portrait*. In this talk, we will illustrate the connection between $J(P)$ and its corresponding critical portrait, and ultimately a critical portrait corresponding to a family of laminations and corresponding family of Julia sets. Two other talks by students will explore combinatorial ways of classifying families of critical portraits, where we introduce the concept of a *weakly bicolored tree* as a classification scheme.