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Critical Portraits of Complex Polynomials

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Abstract: The dynamics of complex polynomials are complicated, and their study is made more manageable through studying less complicated, and more combinatorial, mathematical structures representing families of polynomials with similar dynamical behavior. Families of complex polynomials with similar dynamical behavior can be studied through dynamically equivalent Julia sets. Families of Julia sets can be studied through critical portraits. Families of critical portraits can be represneted by bi-colored and tri-colored trees. The penalty paid for compendious notation is that specificity is lost.

We consider both generic critical portraits in which a full family of non-intersecting critical chords partitions the unit disk into critical sectors, and non-generic critical portraits which contain an all-critical polygon. Weakly bi-colored trees correspond to generic critical portraits. Critical portraits with an all-critical polygon are non-generic but possibly more fundamental. These latter types cannot be represent with weakly bi-colored trees. However, we can represent them as tri-colored trees, where we still have two colors representing fixed and rotational regions, subject to the weekly bi-colored condition, and a third neutral color representing the all-critical polygon(s).

By specifying lamination data, consisting of periodic polygons and a full family of critical chords, we set the stage for a process of "pulling back" the lamination data under branches of the inverse of the angle d-tupling map on the circle to generate a "topological" Julia set. In many cases, the topological Julia set is homeomorphic to an actual Julia set.

In these two talks, John Mayer will discuss the relationship between Julia sets and laminations of the unit disk and introduce the pullback process, illustrated with a managerie of Julia sets. David George will discuss the families of critical portraits and their dynamical equivalence under angle d-tupling and pulling back. Questions about the uniqueness of pullback laminations will be raised.