15th Annual Workshop on Topology and Dynamical Systems
Nipissing University, May 21–25, 2018
http://topology.nipissingu.ca/workshop2018/

Hyperspaces that are cones

Alejandro Illanes (Universidad Nacional Autónoma de México)
vmvm@matem.unam.mx

Abstract: Given a continuum X and a positive integer n, we consider the hyperspaces

- $2^X = \{A : A$ is a nonempty closed subset of $X\}$,
- $C_n(X) = \{A \in 2^X : A$ has at most n components$\}$, and
- $F_n(X) = \{A \in 2^X : A$ has most n points$\}$.

All these hyperspaces are endowed with the Hausdorff metric. As usual, $C_1(X)$ is denoted by $C(X)$. Notice that $F_1(X) = \{\{x\} : x$ is an element of $X\}$ is homeomorphic to X.

The structure of the hyperspace $C(X)$ has some similarities with the cone of X. So it is natural to ask for conditions under which $C(X)$ and cone(X) are homeomorphic. This study has been extended to all hyperspaces. So, given a hyperspace $H(X)$ in $\{2^X, C_n(X), F_n(X)\}$, the general problem in this area is to give conditions on X in such a way that $H(X)$ is the cone over a continuum Z.

In this talk we will give a panoramic view on this problem, from the earlier results to the more recent ones, mentioning also some open questions.