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IMBEDDING COMPACTA INTO CONTINUA[1]

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As we shall show in this paper, a compactum can be imbedded in a continuum in such a manner that certain properties of the components of the compactum are shared by the continuum containing the compactum. A *compactum* is a compact metric space and a *continuum* is a connected compactum.

1. Preliminaries

Bellamy (see [1] and [2]) defines a *pseudocone* to be a Hausdorff compactification S of a half open interval [a,b). Letting $i:[a,b) \to S$ be the injection map, i(a) is the *vertex* of the pseudocone. In addition, if X is homeomorphic to $S \setminus i[a,b)$, then S is a *pseudocone over* X. For $\mathcal{B}$ a collection of sets, we define $|\mathcal{B}| = \bigcup_{C \in \mathcal{B}} C$.

The following lemma was pointed out to the author by A. Lelek.

1.1. *Lemma. Let* X *be a compactum and let* C *be the usual Cantor set. Then* X *can be imbedded in* $C \times X$ *by a function* h *such that* $\{c\} \times X$ *contains at most one component of* h(X) *for* $c \in C$.

Proof. Refer to [15], page 148.

Now suppose X is a compactum. Then letting Q be the Hilbert cube and C be the Cantor set, we can assume without loss of generality that $X \subset C \times Q$ in such a manner that $\{c\} \times Q$ contains at most one component of X for $c \in C$. Define C_0 to be the subset of C such that $c \in C_0$ if and only if $(\{c\} \times Q) \cap X \neq \emptyset$. In addition, define D to be the set of all components of $(M \times Q) \setminus (C_0 \times Q)$ where $M = [-1,2]$.

[1]The author acknowledges his gratitude to H. Cook and A. Lelek for many valuable suggestions.

If $x,y \in M \times Q$, then x is to the *left of* y if and only if the first coordinate of x is less than the first coordinate of y. Also if K and K' are two connected spaces in $M \times Q$, then K is to the left of K' if $x \in K$ and $y \in K'$ implies x is to the left of y. The term "right of" is defined in an analogous manner. Define D_1 and D_2 to be the two elements of D such that D_1 is the left most component of $(M \times Q)\setminus(C_o \times Q)$ and D_2 is the right most component of $(M \times Q)\setminus(C_o \times Q)$. It might be the case that D consists of only two elements. This case occurs when X consists of one component. We will essentially be concerned with the case where D contains an infinite number of elements. For $D_3, D_4, \ldots$, we let $K(L,i)$ be the left component of $\overline{D}_i \cap X$ for $i = 3,4,\ldots$. Similarly, the right component of $\overline{D}_i \cap X$ is denoted $K(R,i)$. Last of all, define $e_n = 4/n$, $n = 1,2,\ldots$, $U(K,e_n) = \{B(x,e_n)\colon x \in K\}$, and $|U(K,e_n)| = \bigcup_{x\in K} B(x,e_n)$.

2. Construction of I (X)

In this section we construct an imbedding of a compactum X into a continuum I(X) by adjoining a countable number of open intervals and two half open intervals in a prescribed manner so as to invariable preserve certain properties of the components of X.

2.1. *Lemma. For a fixed integer* $n > 0$, *there are at most a finite number of* D_i *such that* $|U(K(L,i),e_n)| \cap D_i$ *and* $|U(K(R,i),e_n)| \cap D_i$ *are not contained in the same component of* $|U(X,e_n)| \cap D_i$.

Proof. The lemma is trivially true if X consists of a finite number of components. Thus the lemma is only interesting in the case where X consists of an infinite number of components.

Suppose the lemma is not true for D infinite. In other words, suppose there exists a positive integer N such that there exist an infinite number of D_i such that $|U(K(L,i),e_N)| \cap D_i$ and $|U(K(R,i),e_N)| \cap D_i$ are not contained in the same component of $|U(X,e_N)| \cap D_i$. Let $\{D_1', D_2', \ldots\}$ be such an infinite subset of D. Let $K'(L,i)$ and $K'(R,i)$ be the respective left and right components of X in $\bar{D}_i'$, $i = 1,2,\ldots$. Furthermore let $x'(L,i)$ be an arbitrary point of $K'(L,i)$. Then the set $\{x'(L,i)\}_{i=1}^{\infty}$, contains a limit point $x'(L)$ since X is compact. Let $G(L)$ be the component of X containing $x'(L)$. It follows that for every positive integer n, $B(x'(L),e_n)$ intersects infinitely many $x'(L,i) \in \{x'(L,j)\}_{j=1}^{\infty}$. Choose $x''(L,1)$ from $\{x'(L,i)\}_{i=1}^{\infty}$ such that $x''(L,1) \in B(x'(L),e_1)$, and $x''(L,n)$ from $\{x'(L,i)\}_{i=1}^{\infty}$ $\backslash\{x''(L,i)\}_{i=1}^{n-1}$ such that $x''(L,n) \in B(x'(L),e_n)$ and $d(x'(L),x''(L,n-1)) > d(x'(L),x''(L,n))$, $n = 2,3,\ldots$.

We now define D_i'' such that $x''(L,i) \in \bar{D}_i''$ and $D_i'' \in \{D_1', D_2', \ldots\}$ for all positive integers i. Also $K''(R,i)$ is defined to be the right component of $\bar{D}_i'' \cap X$, and $x''(R,i)$ is defined to be an arbitrary point of $K''(R,i)$ for all positive integers i. The set of points $\{x''(R,i)\}_{i=1}^{\infty}$ is an infinite set of points belonging to the compact space X. Hence the set of points has a limit point, say $x''(R)$, where $x''(R) \in X$. Thus for every positive integer n, $B(x''(R),e_n)$ intersects infinitely many $x''(R,i) \in \{x''(R,j)\}_{j=1}^{\infty}$. Choose $x'''(R,1) \in \{x''(R,j)\}_{j=1}^{\infty}$. Also choose $x'''(R,n) \in B(x''(R),e_n)$ satisfying $x'''(R,n) \in \{x''(R,j)\}_{j=1}^{\infty}\backslash\{x'''(R,j)\}_{j=1}^{n-1}$, and such that $x'''(R,n-1) = x''(R,j_{n-1})$ and $x'''(R,n) = x''(R,j_n)$ implies $j_{n-1} < j_n$ for all integers n greater than one. Define $D_i''' \in \{D_j''\}_{j=1}^{\infty}$ such that $x'''(R,i) \in \bar{D}_i'''$. It follows that for all integers N' greater than N, there exists a positive integer m satisfying the inequalities $d(x'(L),x'''(L,m)) < 1/N'$ and $d(x''(R),x'''(R,m)) < 1/N'$.

It is not hard to show that $x'(L)$ and $x''(R)$ have the same first coordinate. Thus if $G(R)$ is the component of X such that $x''(R)$ belongs to $G(R)$, then $G(R) = G(L)$. This implies

$$B(x'(L),e_N) \cap |U(K'''(L,m),e_N)| \cap D_m''' \neq \emptyset,$$

and

$$B(x''(R),e_N) \cap |U(K'''(R,m),e_N)| \cap D_m''' \neq \emptyset.$$

Consequently $|U(K'''(L,m),e_N)| \cap D_m'''$ and $|U(K'''(R,m),e_N)| \cap D_m'''$ are contained in the same component of the open set $|U(X,e_N)| \cap D_m'''$. This is a contradiction since D_m''' belongs to $\{D_i'\}_{i=1}^{\infty}$.

For the next two lemmas, we use the following notation. Let I' be a closed interval in the interval [-1,2]. Define X' to be $X \cap (I' \times Q)$ and D(I') to be the set of components of $(I' \times Q)\backslash(C_o \times Q)$.

2.2. *Lemma. For a fixed integer* $n > 0$, *there are at most a finite number of* D_i *such that* $|U(K(L,i),e_n)| \cap D_i$ *and* $|U(K(R,i)e_n)| \cap D_i$ *are not contained in the same component of* $|U(X',e_n)| \cap D_i$ *for* $D_i \in D(I')$.

The above lemma is the same as 2.1 only with a notational change.

2.3. *Lemma. If* N *is a fixed positive integer and for all* D_i *in* D(I'), $|U(K(L,i),e_N)| \cap D_i$ *and* $|U(K(R,i),e_N)| \cap D_i$ *are contained in the same component of* $|U(X',e_N)| \cap D_i$, *then* $\overline{|U(X',e_N)|} \cap (I' \times Q)$ *is a continuum containing* X'.

The proof is left to the reader.

2.4. *Lemma. Let* N *be a fixed positive integer,* D_i *be an element of* D, *and* K_i *be a component of* X *in* $\overline{D}_i$. *Then in* $|U(K_i,e_N)| \cap \overline{D}_i$ *there is a pseudocone* S *over* K_i *such that*

$$(\text{bdry } D_i) \cap S = K_i.$$

The proof is essentially the same as that presented for Lemma 3 of [1].

2.5. *Lemma. Let* N *be a fixed positive integer and* D_i *be such that* $|U(K(L,i),e_N)| \cap D_i$ *and* $|U(K(R,i),e_N)| \cap D_i$ *are* *contained in the same component of* $|U(X',e_N)| \cap D_i$. *Then there is a homeomorphism* $h_i: (-\infty,\infty) \to [|U(X',e_N)| \cap D_i]$ *such that*

$$\overline{h_i((-\infty,\infty))} = K(L,i) \cup h((-\infty,\infty)) \cup K(R,i).$$

Proof. Lemma 2.4 is used to get two disjoint pseudocones, one over K(L,i) and the other over K(R,i). This is done so that the union of the pseudocones intersected with the boundary of D_i is $K(L,i) \cup K(R,i)$. It is a simple matter to join the two vertices of the pseudocones with an arc in the component of $|U(X,e_N)| \cap D_i$ containing $|U(K(L,i),e_N)| \cap D_i$ and $|U(K(R,i),e_N)| \cap D_i$ such that the arc minus its end points does not intersect the two pseudocones.

In the future, we will refer to $h((-\infty,\infty))$ as a *connector*.

2.6. *Theorem. Given any compactum* X *as described in section* 1, *there is a continuum* I(X) *such that two components of* $I(X)\setminus X$ *are homeomorphic to half open intervals and the remaining components are homeomorphic to open intervals. Furthermore*

$$I(X) = \overline{I(X)\setminus X},$$

and for $c \in [-1,2]\setminus C_o$, $\{c\} \times Q$ *intersects at most one of the two half open intervals or one of the open intervals. It does not, however, intersect a half open interval and an open interval.*

Proof. In $\overline{D}_1$ and $\overline{D}_2$, we construct two pseudocones over K(R,1) and K(L,2) as described in 2.4. If X contains only a finite number of components, then in each of the remaining $D_i \in D$, we construct connectors as described in 2.5. The union of X, the two pseudocones, and the connectors, is a continuum satisfying the theorem.

Suppose X consists of an infinite number of components. We first recall that $e_1 = 4$ and for $x,y \in X$, $d(x,y) \leq 2$. In fact the above inequality holds for $x,y \in I \times Q$ where $I = [0,1]$.

We note one other thing, namely that $\overline{|U(X,e_1)|} \cap (I \times Q) = I \times Q$. Thus the continuum A_1', consisting of the two pseudocones unioned with $I \times Q$, contains X.

We will define A_n inductively. Suppose A_{n-1} is a continuum in $M \times Q$ such that A_{n-1} contains two pseudocones, as described in 2.4, in $\overline{D}_1$ and $\overline{D}_2$. Also A_{n-1} contains a finite number of connectors as described in 2.5. Let $D_3',\ldots,D_{d_{n-1}}'$ be the elements of D corresponding to the connectors of A_{n-1}. Let $I_1,\ldots,I_{k_{n-1}}$ be the largest closed intervals in I such that $I_j \times Q$ does not intersect a connector. We assume that for $N = i_{n-1}$ and D_i in $(I_1 \times Q) \cup \ldots \cup (I_{k_{n-1}} \times Q)$ where $D_i \in D$, the hypothesis of 2.5 is satisfied. In addition, suppose A_{n-1} contains the components of $\overline{|U(X,e_{i_{n-1}})|} \cap (I_j \times Q)$ which contain $X \cap (I_j \times Q)$. Call these components H_j, $j = 1,\ldots,k_{n-1}$. The continuum A_{n-1} is the union of two pseudocones, a finite number of connectors, and continua $H_1,\ldots,H_{k_{n-1}}$ as described above. Furthermore, two of the components of $A_{n-1}\backslash(H_1 \cup \ldots \cup H_{k_{n-1}})$ are homeomorphic to half open intervals while the remaining components are homeomorphic to open intervals. We define $A_{n-1} = A_1'$ and $e_{i_{n-1}} = e_1$ for $n = 2$. We will now define A_n assuming A_{n-1} is known.

We choose an integer i_n greater than i_{n-1} such that for at least one $D_i \in D$, $D_i \cap (I_j \times Q) \neq \emptyset$, $|U(K(L,i),e_{i_n})| \cap D_i$ and $|U(K(R,i),e_{i_n})| \cap D_i$ are not contained in the same component of $|U(X,e_{i_n})| \cap (I_j \times Q)$ where $j \in \{1,2,\ldots,k_{n-1}\}$. By 2.2, there are at most a finite number of $D_i \in D$, say $D_{d_{n-1}+1}',\ldots,D_{d_n}'$, with the above property. If $\alpha \in \{d_{n-1}+1,d_{n-1}+2,\ldots,d_n\}$, then there is a $j \in \{1,\ldots,k_{n-1}\}$ such that $D_\alpha' \cap (I_j \times Q) \neq \emptyset$. For each such D' there is by 2.5 a connector in the component of $|U(X,e_{i_{n-1}})| \cap D_\alpha'$ containing $|U(K(L,i),e_{i_{n-1}})| \cap D_\alpha'$ and $|U(K(R,i),e_{i_{n-1}})| \cap D_\alpha'$. These new connectors are in $H_1 \cup \ldots \cup H_{k_{n-1}}$. Let $I_1',\ldots,I_{k_n}'$ be the largest closed intervals in I such that $I_j' \times Q$ does not intersect a connector of A_{n-1}, a new connector, or the half open intervals of the pseudocones. Thus for $N = i_n$ and D_i in $(I_1' \times Q) \cup \ldots \cup (I_{k_n}' \times Q)$ where $D_i \in D$, the hypothesis of 2.5 is satisfied. Let A_n contain the components of $\overline{|U(X,e_{i_n})|} \cap (I_j' \times Q)$ which contain $X \cap (I_j' \times Q)$, $j = 1,2,\ldots,k_n$. Then the continuum A_n is the union of two pseudocones (the ones from A_{n-1}), a finite number of connectors (the ones from A_{n-1} plus the new connectors defined above), and continua $H_1',\ldots,H_{k_n}'$ where H_j' is the component of $\overline{|U(X,e_{i_n})|} \cap (I_j' \times Q)$ mentioned above, $j = 1,\ldots,k_n$. The continuum A_n has the property that two of the components of $A_n \backslash (H_1' \cup \ldots \cup K_{k_n}')$ are homeomorphic to the half open intervals while the remaining components are homeomorphic to open intervals.

The half open intervals of the pseudocones of A_n and the connectors of A_n are such that for $c \in [-1,2]\backslash C_o$, if $\{c\} \times Q$ intersects a connector, then $\{c\} \times Q$ intersects at most one connector and does not intersect a half open interval of a pseudocone of A_n. If $\{c\} \times Q$ intersects a half open interval of a pseudocone of A_n, then $\{c\} \times Q$ intersects only one of the half open intervals of the pseudocones and does not intersect any of the connectors of A_n.

One of the properties of A_n, $n = 1,2,\ldots$, is

$$A_1 \supset A_2 \supset A_3 \supset \ldots.$$

We define $I(X) = \bigcap_{i=1}^{\infty} A_i$. It is obvious that $I(X)$ has the properties described in Theorem 2.6.

We will call the half open interval of the left pseudocone, the *left connector* and the half open interval of the right pseudocone, the *right connector*.

3. Properties of I (X) relative to the components of X

We show in this section that $I(X)$ preserves many properties possessed by the components of X provided all of the components of X have these properties.

We first state the theorem on which all of the remaining results of this section depend.

3.1. *Theorem. There is a monotone mapping* f *of* I(X) *onto* M *such that* $f:(I(X)\backslash X) \to (M\backslash C_o)$ *is a homeomorphism and point inverses of* C_o *are components of* X.

The proof of this theorem is straightforward. In fact, the proofs of many of the following theorems are straightforward and thus are left to the reader.

3.2. *Theorem. If* X *is any compactum, then* I(X) *is irreducible between two of its points.*

We list a few definitions that will be used in the theorems which follow.

We say a continuum X is *acyclic* if every mapping of X into the circle in homotopic to a constant mapping. Given that each collection of mutually disjoint nondegenerate subcontinua of a continuum X is countable, we say that X is *Suslinian*. A continuum is *rational* if it has a basis of open sets with countable boundaries. If α is an open cover of the compactum X, a map f of X onto a compactum Y is called an α-*map* provided that for each y in Y, $f^{-1}(y)$ is contained in some member of α. A *tree* is a 1-dimensional acyclic connected graph. A continuum X is *tree-like* if for each open cover α of X, there is an α-map of X onto some tree. *Arc-like* is defined in a similar manner.

3.3. *Theorem. If* X *is any compactum, then* I(X) *is decomposable.*

3.4. *Theorem. If* X *is a compactum and each nondegenerate component is hereditarily decomposable, then* I(X) *is hereditarily decomposable.*

3.5. *Theorem. If* X *is a compactum, then* I(X) *is unicoherent.*

Proof. By 3.3, I(X) is decomposable. If A and B are two proper subcontinua of I(X) such that $I(X) = A \cup B$, then either A or B must contain the end point of the left connector. Without loss of generality, we assume A contains the end point. Then f(A) contains the point -1. It follows that f(B) contains 2, but not -1. Since A is connected and nontrivial, f(A) is an interval [-1,a] in M. For the same reasons f(B) is an interval [b,2] in M. Thus $f(A \cap B) = [b,a]$ or $A \cap B = f^{-1}([b,a])$, either of which is a continuum.

For A or B a point, the proof is trivial.

3.6. *Theorem. If* X *is a compactum and each component of* X *is hereditarily unicoherent, then* I(X) *is hereditarily unicoherent.*

The proof is similar to the proof of 3.5.

3.7. *Theorem. If* X *is a compactum and each nondegenerate component is a* λ*-dendroid, then* I(X) *is a* λ*-dendroid.*

Proof. Since a λ-dendroid is by definition a hereditarily unicoherent, hereditarily decomposable continuum, 3.7 follows directly from 3.4 and 3.6.

3.8. *Theorem. If* X *is a compactum and each component is atriodic, then* I(X) *is atriodic.*

3.9. *Theorem. If* X *is a compactum and each component is either a point or an arc-like continuum, then* I(X) *is an arc-like continuum.*

Proof. Notice that each component of X is hereditarily unicoherent and atriodic. Thus by 3.6 and 3.8, I(X) is hereditarily unicoherent and atriodic. If a nondegenerate subcontinuum Y of I(X) is indecomposable, then Y is a subcontinuum of a component of X. In this case Y is arc-like. Thus I(X) is hereditarily unicoherent, atriodic, and every nontrivial indecomposable subcontinuum of I(X) is arc-like. By Theorem 2 of [9], I(X) is arc-like.

Remarks. The reader should compare 3.9 with Theorem 2.1 of [16], and Theorem 11 of [5]. Theorem 3.9 answers the question posed by A. Lelek in a conversation with the author, "Can every compactum whose components are arc-like be imbedded in an arc-like continuum?"

3.10. *Corollary. If* X *is a compactum and each nondegenerate component of* X *is arc-like, then* X *is planar.*

Proof. Bing [3] has proved that every arc-like continuum is planar. This and 3.9 yield the corollary.

3.11. *Theorem. If* X *is a compactum and each component is either a point or a tree-like continuum, then* I(X) *is a tree-like continuum.*

Proof. Again we note that each component of X is hereditarily unicoherent. Thus I(X) is hereditarily unicoherent by 3.6. If Y is a nontrivial indecomposable subcontinuum of I(X), then Y is a subcontinuum of a component of X. Since every subcontinuum of a tree-like continuum is tree-like, Y is tree-like. Hence I(X) is hereditarily unicoherent and every nontrivial indecomposable subcontinuum of I(X) is tree-like. By Theorem 1 of [6], I(X) is tree-like.

3.12. *Theorem. If* X *is a compactum and each component is of dimension at most* k $(k > 0)$, *then* I(X) *is of dimension at most* k.

Proof. The continuum I(X) is the union of X and $I(X)\setminus X$. The set $I(X)\setminus X$ consists of a countable number of components each of which is homeomorphic to either an open interval or a half open interval. Thus $I(X)\setminus X$ is the countable union of arcs. We note that X is of dimension at most k $(k > 0)$. From

Theorem III 2 of [10], we get our theorem.

3.13. *Theorem. If* X *is a compactum and each component is acyclic, then* I(X) *is acyclic.*

Proof. Let g be a mapping of I(X) into the circle. The mapping is either essential or it is not. If the mapping is not essential, then it is inessential and in this case is homotopic to a constant mapping.

If g is an essential mapping of I(X) into the circle S^1, then there exists in I(X) a continuum K with the property that $g|K$ is not homotopic to a constant mapping, but every proper subcontinuum K' of K is such that $g|K'$ is homotopic to a constant mapping. Furthermore, K is discoherent ([4], p. 216). The continuum K is *discoherent* if the complement of each subcontinuum of K is connected (refer to [15], p. 163). If f(K) is not a point, then K is unicoherent. This we have proved earlier. Thus K must be a subcontinuum of one of the components of X. Since $g|K$ is not homotopic to a constant mapping, g restricted to the component N of X containing K is not homotopic to a constant mapping. It follows that N is not acyclic. This contradicts the fact that N is acyclic.

Since g is not an essential mapping, g is an inessential mapping. Also since g was arbitrary, we have that I(X) is acyclic.

3.14. *Theorem. If* X *is a compactum where each component is rational, and* X *contains at most a countable number of non-degenerate components, then* I(X) *is rational.*

3.15. *Theorem. If* X *is a compactum where each component is Suslinian, and* X *contains at most a countable number of non-degenerate components, then* I(X) *is Suslinian.*

Remarks. There might be times when one would want to imbed a compactum as described in the hypothesis of 3.15 in a locally connected Suslinian curve. Fitzpatrick and Lelek describe such an imbedding in [8]. It follows from their work that subcompacta of Suslinian continua are characterized by those properties described in the hypothesis of 3.15.

4. Characteristics of subcompacta of specific curves

Recall that a *curve* is a 1-dimensional continuum.

In this section we include some of the more immediate results which follow from our work in section 3.

4.1. *Theorem. A compactum* X *can be imbedded in a rational curve if and only if* X *contains at most a countable number of nontrivial components and each component is rational.*

This theorem follows directly from 3.14 and Theorem 5, page 285, in [15].

4.2. *Theorem. A compactum* X *can be imbedded in an acyclic curve if and only if the components of* X *are either degenerate or acyclic curves.*

Proof. If Y is an acyclic curve, then every subcontinuum of Y is an acyclic curve.

By 3.12 and 3.13, if every component of X is acyclic and at most 1-dimensional, then X can be imbedded in an acyclic 1-dimensional continuum.

4.3. *Theorem. A compactum* X *can be imbedded in an atriodic tree-like curve if and only if each nondegenerate component of* X *is atriodic and tree-like.*

The proof follows immediately from 3.8 and 3.11.

4.4. *Corollary. Either there exist nonplanar atriodic tree-like curves, or given any atriodic tree-like curve* X*, the plane contains uncountably many mutually disjoint homeomorphic* *copies of* X.

Proof. Let X be an atriodic tree-like curve. Either X is nonplanar or it is not. If it is nonplanar, then the theorem is true. If X is planar, then $C \times X$, where C is the usual Cantor set, is a compactum and each component is atriodic and tree-like. Hence $C \times X$ can be imbedded in an atriodic tree-like curve Y by 4.3. Either Y is planar or it is not. If Y is planar, then the plane contains uncountably many mutually disjoint homeomorphic copies of X. If Y is not planar, then the first part of the theorem is true.

Remarks. This result relates two questions. Bing in [3] asks if given any atriodic tree-like planar curve X, does the plane contain uncountably many mutually disjoint homeomorphic copies of X? Ingram [11] proved the existence of an atriodic tree-like curve in the plane which is not arc-like. Furthermore in [12], Ingram proved that uncountably many atriodic tree-like continua, none of which is arc-like, can be imbedded in the plane so that they are mutually exclusive. Ingram, at this conference, asked whether there exist nonplanar atriodic tree-like curves.

4.5. *Theorem. A compactum* X *can be imbedded in a hereditarily decomposable continuum if and only if each nondegenerate component of* X *is hereditarily decomposable.*

From 3.4 and the definition of hereditarily decomposable, the theorem easily follows.

4.6. *Theorem. A compactum* X *can be imbedded in a hereditarily unicoherent curve if and only if each nondegenerate component of* X *is a hereditarily unicoherent curve.*

Theorem 4.6 follows directly from 3.6 and the definition of hereditarily unicoherent.

4.7. *Theorem. A compactum* X *can be imbedded in a* λ*-dendroid if and only if each nondegenerate component of* X *is a* λ*-dendroid.*

This theorem follows directly from 3.7 and the definition of λ-dendroid.

Remarks. It is evident that we did not exhaust the possible results in this section. However, it is clear how one would proceed in order to get similar results.

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