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IMBEDDING COMPACTA INTO CONTINUA¹

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As we shall show in this paper, a compactum can be imbedded in a continuum in such a manner that certain properties of the components of the compactum are shared by the continuum containing the compactum. A *compactum* is a compact metric space and a *continuum* is a connected compactum.

1. Preliminaries

Bellamy (see [1] and [2]) defines a *pseudocone* to be a Hausdorff compactification S of a half open interval $[a,b)$. Letting $i:[a,b) \rightarrow S$ be the injection map, $i(a)$ is the *vertex* of the pseudocone. In addition, if X is homeomorphic to $S \setminus i[a,b)$, then S is a *pseudocone over X* . For \mathcal{B} a collection of sets, we define $|\mathcal{B}| = \bigcup_{C \in \mathcal{B}} C$.

The following lemma was pointed out to the author by A. Lelek.

1.1. *Lemma.* *Let X be a compactum and let C be the usual Cantor set. Then X can be imbedded in $C \times X$ by a function h such that $\{c\} \times X$ contains at most one component of $h(X)$ for $c \in C$.*

Proof. Refer to [15], page 148.

Now suppose X is a compactum. Then letting Q be the Hilbert cube and C be the Cantor set, we can assume without loss of generality that $X \subset C \times Q$ in such a manner that $\{c\} \times Q$ contains at most one component of X for $c \in C$. Define C_0 to be the subset of C such that $c \in C_0$ if and only if $(\{c\} \times Q) \cap X \neq \emptyset$. In addition, define D to be the set of all components of $(M \times Q) \setminus (C_0 \times Q)$ where $M = [-1,2]$.

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If $x, y \in M \times Q$, then x is to the *left of* y if and only if the first coordinate of x is less than the first coordinate of y . Also if K and K' are two connected spaces in $M \times Q$, then K is to the left of K' if $x \in K$ and $y \in K'$ implies x is to the left of y . The term "right of" is defined in an analogous manner. Define D_1 and D_2 to be the two elements of D such that D_1 is the left most component of $(M \times Q) \setminus (C_0 \times Q)$ and D_2 is the right most component of $(M \times Q) \setminus (C_0 \times Q)$. It might be the case that D consists of only two elements. This case occurs when X consists of one component. We will essentially be concerned with the case where D contains an infinite number of elements. For D_3, D_4, \dots , we let $K(L, i)$ be the left component of $\bar{D}_i \cap X$ for $i = 3, 4, \dots$. Similarly, the right component of $\bar{D}_i \cap X$ is denoted $K(R, i)$. Last of all, define $e_n = 4/n$, $n = 1, 2, \dots$, $U(K, e_n) = \{B(x, e_n) : x \in K\}$, and $|U(K, e_n)| = \bigcup_{x \in K} B(x, e_n)$.

2. Construction of $I(X)$

In this section we construct an imbedding of a compactum X into a continuum $I(X)$ by adjoining a countable number of open intervals and two half open intervals in a prescribed manner so as to invariable preserve certain properties of the components of X .

2.1. *Lemma.* For a fixed integer $n > 0$, there are at most a finite number of D_i such that $|U(K(L, i), e_n)| \cap D_i$ and $|U(K(R, i), e_n)| \cap D_i$ are not contained in the same component of $|U(X, e_n)| \cap D_i$.

Proof. The lemma is trivially true if X consists of a finite number of components. Thus the lemma is only interesting in the case where X consists of an infinite number of components.

Suppose the lemma is not true for D infinite. In other words, suppose there exists a positive integer N such that

there exist an infinite number of D_i such that $|U(K(L,i), e_N)| \cap D_i$ and $|U(K(R,i), e_N)| \cap D_i$ are not contained in the same component of $|U(X, e_N)| \cap D_i$. Let $\{D'_1, D'_2, \dots\}$ be such an infinite subset of D . Let $K'(L,i)$ and $K'(R,i)$ be the respective left and right components of X in \bar{D}'_i , $i = 1, 2, \dots$. Furthermore let $x'(L,i)$ be an arbitrary point of $K'(L,i)$. Then the set $\{x'(L,i)\}_{i=1}^\infty$, contains a limit point $x'(L)$ since X is compact. Let $G(L)$ be the component of X containing $x'(L)$. It follows that for every positive integer n , $B(x'(L), e_n)$ intersects infinitely many $x'(L,i) \in \{x'(L,j)\}_{j=1}^\infty$. Choose $x''(L,1)$ from $\{x'(L,i)\}_{i=1}^\infty$ such that $x''(L,1) \in B(x'(L), e_1)$, and $x''(L,n)$ from $\{x'(L,i)\}_{i=1}^\infty \setminus \{x''(L,i)\}_{i=1}^{n-1}$ such that $x''(L,n) \in B(x'(L), e_n)$ and $d(x'(L), x''(L, n-1)) > d(x'(L), x''(L, n))$, $n = 2, 3, \dots$.

We now define D''_i such that $x''(L,i) \in \bar{D}''_i$ and $D''_i \in \{D'_1, D'_2, \dots\}$ for all positive integers i . Also $K''(R,i)$ is defined to be the right component of $\bar{D}''_i \cap X$, and $x''(R,i)$ is defined to be an arbitrary point of $K''(R,i)$ for all positive integers i . The set of points $\{x''(R,i)\}_{i=1}^\infty$ is an infinite set of points belonging to the compact space X . Hence the set of points has a limit point, say $x''(R)$, where $x''(R) \in X$. Thus for every positive integer n , $B(x''(R), e_n)$ intersects infinitely many $x''(R,i) \in \{x''(R,j)\}_{j=1}^\infty$. Choose $x'''(R,1) \in \{x''(R,j)\}_{j=1}^\infty$. Also choose $x'''(R,n) \in B(x''(R), e_n)$ satisfying $x'''(R,n) \in \{x''(R,j)\}_{j=1}^\infty \setminus \{x'''(R,j)\}_{j=1}^{n-1}$, and such that $x'''(R, n-1) = x''(R, j_{n-1})$ and $x'''(R, n) = x''(R, j_n)$ implies $j_{n-1} < j_n$ for all integers n greater than one. Define $D'''_i \in \{D''_j\}_{j=1}^\infty$ such that $x'''(R,i) \in \bar{D}'''_i$. It follows that for all integers N' greater than N , there exists a positive integer m satisfying the inequalities $d(x'(L), x'''(L, m)) < 1/N'$ and $d(x''(R), x'''(R, m)) < 1/N'$.

It is not hard to show that $x'(L)$ and $x''(R)$ have the same first coordinate. Thus if $G(R)$ is the component of X such that $x''(R)$ belongs to $G(R)$, then $G(R) = G(L)$. This implies

$$B(x'(L), e_N) \cap |U(K'''(L, m), e_N)| \cap D'''_m \neq \emptyset,$$

and

$$B(x''(R), e_N) \cap |U(K'''(R, m), e_N)| \cap D_m''' \neq \emptyset.$$

Consequently $|U(K'''(L, m), e_N)| \cap D_m'''$ and $|U(K'''(R, m), e_N)| \cap D_m'''$ are contained in the same component of the open set $|U(X, e_N)| \cap D_m'''$. This is a contradiction since D_m''' belongs to $\{D_i'\}_{i=1}^\infty$.

For the next two lemmas, we use the following notation. Let I' be a closed interval in the interval $[-1, 2]$. Define X' to be $X \cap (I' \times Q)$ and $D(I')$ to be the set of components of $(I' \times Q) \setminus (C_0 \times Q)$.

2.2. *Lemma.* For a fixed integer $n > 0$, there are at most a finite number of D_i such that $|U(K(L, i), e_n)| \cap D_i$ and $|U(K(R, i), e_n)| \cap D_i$ are not contained in the same component of $|U(X', e_n)| \cap D_i$ for $D_i \in D(I')$.

The above lemma is the same as 2.1 only with a notational change.

2.3. *Lemma.* If N is a fixed positive integer and for all D_i in $D(I')$, $|U(K(L, i), e_N)| \cap D_i$ and $|U(K(R, i), e_N)| \cap D_i$ are contained in the same component of $|U(X', e_N)| \cap D_i$, then $\overline{|U(X', e_N)|} \cap (I' \times Q)$ is a continuum containing X' .

The proof is left to the reader.

2.4. *Lemma.* Let N be a fixed positive integer, D_i be an element of D , and K_i be a component of X in \bar{D}_i . Then in $|U(K_i, e_N)| \cap \bar{D}_i$ there is a pseudocone S over K_i such that $(\text{bdry } D_i) \cap S = K_i$.

The proof is essentially the same as that presented for Lemma 3 of [1].

2.5. *Lemma.* Let N be a fixed positive integer and D_i be such that $|U(K(L, i), e_N)| \cap D_i$ and $|U(K(R, i), e_N)| \cap D_i$ are

contained in the same component of $|U(X', e_N) \cap D_i$. Then there is a homeomorphism $h_i: (-\infty, \infty) \rightarrow [|U(X', e_N) \cap D_i|]$ such that

$$h_i(\overline{(-\infty, \infty)}) = K(L, i) \cup h((-\infty, \infty)) \cup K(R, i).$$

Proof. Lemma 2.4 is used to get two disjoint pseudocones, one over $K(L, i)$ and the other over $K(R, i)$. This is done so that the union of the pseudocones intersected with the boundary of D_i is $K(L, i) \cup K(R, i)$. It is a simple matter to join the two vertices of the pseudocones with an arc in the component of $|U(X, e_N) \cap D_i$ containing $|U(K(L, i), e_N) \cap D_i$ and $|U(K(R, i), e_N) \cap D_i$ such that the arc minus its end points does not intersect the two pseudocones.

In the future, we will refer to $h((-\infty, \infty))$ as a *connector*.

2.6. *Theorem.* Given any compactum X as described in section 1, there is a continuum $I(X)$ such that two components of $I(X) \setminus X$ are homeomorphic to half open intervals and the remaining components are homeomorphic to open intervals. Furthermore

$$I(X) = \overline{I(X) \setminus X},$$

and for $c \in [-1, 2] \setminus C_0$, $\{c\} \times Q$ intersects at most one of the two half open intervals or one of the open intervals. It does not, however, intersect a half open interval and an open interval.

Proof. In \bar{D}_1 and \bar{D}_2 , we construct two pseudocones over $K(R, 1)$ and $K(L, 2)$ as described in 2.4. If X contains only a finite number of components, then in each of the remaining $D_i \in D$, we construct connectors as described in 2.5. The union of X , the two pseudocones, and the connectors, is a continuum satisfying the theorem.

Suppose X consists of an infinite number of components. We first recall that $e_1 = 4$ and for $x, y \in X$, $d(x, y) \leq 2$. In fact the above inequality holds for $x, y \in I \times Q$ where $I = [0, 1]$.

We note one other thing, namely that $|U(X, e_1)| \cap (I \times Q) = I \times Q$. Thus the continuum A'_1 , consisting of the two pseudocones unioned with $I \times Q$, contains X .

We will define A_n inductively. Suppose A_{n-1} is a continuum in $M \times Q$ such that A_{n-1} contains two pseudocones, as described in 2.4, in \bar{D}_1 and \bar{D}_2 . Also A_{n-1} contains a finite number of connectors as described in 2.5. Let $D'_3, \dots, D'_{d_{n-1}}$ be the elements of D corresponding to the connectors of A_{n-1} . Let $I_1, \dots, I_{k_{n-1}}$ be the largest closed intervals in I such that $I_j \times Q$ does not intersect a connector. We assume that for $N = i_{n-1}$ and D_i in $(I_1 \times Q) \cup \dots \cup (I_{k_{n-1}} \times Q)$ where $D_i \in D$, the hypothesis of 2.5 is satisfied. In addition, suppose A_{n-1} contains the components of $\overline{|U(X, e_{i_{n-1}})|} \cap (I_j \times Q)$ which contain $X \cap (I_j \times Q)$. Call these components H_j , $j = 1, \dots, k_{n-1}$. The continuum A_{n-1} is the union of two pseudocones, a finite number of connectors, and continua $H_1, \dots, H_{k_{n-1}}$ as described above. Furthermore, two of the components of $A_{n-1} \setminus (H_1 \cup \dots \cup H_{k_{n-1}})$ are homeomorphic to half open intervals while the remaining components are homeomorphic to open intervals. We define $A_{n-1} = A'_1$ and $e_{i_{n-1}} = e_1$ for $n = 2$. We will now define A_n assuming A_{n-1} is known.

We choose an integer i_n greater than i_{n-1} such that for at least one $D_i \in D$, $D_i \cap (I_j \times Q) \neq \emptyset$, $|U(K(L, i), e_{i_n})| \cap D_i$ and $|U(K(R, i), e_{i_n})| \cap D_i$ are not contained in the same component of $|U(X, e_{i_n})| \cap (I_j \times Q)$ where $j \in \{1, 2, \dots, k_{n-1}\}$. By 2.2, there are at most a finite number of $D_i \in D$, say $D'_{d_{n-1}+1}, \dots, D'_{d_n}$, with the above property. If $\alpha \in \{d_{n-1}+1, d_{n-1}+2, \dots, d_n\}$, then there is a $j \in \{1, \dots, k_{n-1}\}$ such that $D'_\alpha \cap (I_j \times Q) \neq \emptyset$. For each such D' there is by 2.5 a connector in the component of $|U(X, e_{i_{n-1}})| \cap D'_\alpha$ containing $|U(K(L, i), e_{i_{n-1}})| \cap D'_\alpha$ and $|U(K(R, i), e_{i_{n-1}})| \cap D'_\alpha$. These new connectors are in $H_1 \cup \dots \cup H_{k_{n-1}}$. Let I'_1, \dots, I'_{k_n} be the largest closed intervals in I such that $I'_j \times Q$ does not intersect a connector

of A_{n-1} , a new connector, or the half open intervals of the pseudocones. Thus for $N = i_n$ and D_i in $(I'_1 \times Q) \cup \dots \cup (I'_{k_n} \times Q)$ where $D_i \in D$, the hypothesis of 2.5 is satisfied. Let A_n contain the components of $\overline{|U(X, e_{i_n})|} \cap (I'_j \times Q)$ which contain $X \cap (I'_j \times Q)$, $j = 1, 2, \dots, k_n$. Then the continuum A_n is the union of two pseudocones (the ones from A_{n-1}), a finite number of connectors (the ones from A_{n-1} plus the new connectors defined above), and continua H'_1, \dots, H'_{k_n} where H'_j is the component of $\overline{|U(X, e_{i_n})|} \cap (I'_j \times Q)$ mentioned above, $j = 1, \dots, k_n$. The continuum A_n has the property that two of the components of $A_n \setminus (H'_1 \cup \dots \cup K'_{k_n})$ are homeomorphic to the half open intervals while the remaining components are homeomorphic to open intervals.

The half open intervals of the pseudocones of A_n and the connectors of A_n are such that for $c \in [-1, 2] \setminus C_0$, if $\{c\} \times Q$ intersects a connector, then $\{c\} \times Q$ intersects at most one connector and does not intersect a half open interval of a pseudocone of A_n . If $\{c\} \times Q$ intersects a half open interval of a pseudocone of A_n , then $\{c\} \times Q$ intersects only one of the half open intervals of the pseudocones and does not intersect any of the connectors of A_n .

One of the properties of A_n , $n = 1, 2, \dots$, is

$$A_1 \supset A_2 \supset A_3 \supset \dots$$

We define $I(X) = \bigcap_{i=1}^{\infty} A_i$. It is obvious that $I(X)$ has the properties described in Theorem 2.6.

We will call the half open interval of the left pseudocone, the *left connector* and the half open interval of the right pseudocone, the *right connector*.

3. Properties of $I(X)$ relative to the components of X

We show in this section that $I(X)$ preserves many properties possessed by the components of X provided all of the components of X have these properties.

We first state the theorem on which all of the remaining results of this section depend.

3.1. *Theorem.* *There is a monotone mapping f of $I(X)$ onto M such that $f: (I(X) \setminus X) \rightarrow (M \setminus C_0)$ is a homeomorphism and point inverses of C_0 are components of X .*

The proof of this theorem is straightforward. In fact, the proofs of many of the following theorems are straightforward and thus are left to the reader.

3.2. *Theorem.* *If X is any compactum, then $I(X)$ is irreducible between two of its points.*

We list a few definitions that will be used in the theorems which follow.

We say a continuum X is *acyclic* if every mapping of X into the circle is homotopic to a constant mapping. Given that each collection of mutually disjoint nondegenerate subcontinua of a continuum X is countable, we say that X is *Suslinian*. A continuum is *rational* if it has a basis of open sets with countable boundaries. If α is an open cover of the compactum X , a map f of X onto a compactum Y is called an α -map provided that for each y in Y , $f^{-1}(y)$ is contained in some member of α . A *tree* is a 1-dimensional acyclic connected graph. A continuum X is *tree-like* if for each open cover α of X , there is an α -map of X onto some tree. *Arc-like* is defined in a similar manner.

3.3. *Theorem.* *If X is any compactum, then $I(X)$ is decomposable.*

3.4. *Theorem.* *If X is a compactum and each nondegenerate component is hereditarily decomposable, then $I(X)$ is hereditarily decomposable.*

3.5. *Theorem.* *If X is a compactum, then $I(X)$ is unicoherent.*

Proof. By 3.3, $I(X)$ is decomposable. If A and B are two proper subcontinua of $I(X)$ such that $I(X) = A \cup B$, then either A or B must contain the end point of the left connector. Without loss of generality, we assume A contains the end point. Then $f(A)$ contains the point -1 . It follows that $f(B)$ contains 2 , but not -1 . Since A is connected and nontrivial, $f(A)$ is an interval $[-1, a]$ in M . For the same reasons $f(B)$ is an interval $[b, 2]$ in M . Thus $f(A \cap B) = [b, a]$ or $A \cap B = f^{-1}([b, a])$, either of which is a continuum.

For A or B a point, the proof is trivial.

3.6. *Theorem.* If X is a compactum and each component of X is hereditarily unicoherent, then $I(X)$ is hereditarily unicoherent.

The proof is similar to the proof of 3.5.

3.7. *Theorem.* If X is a compactum and each nondegenerate component is a λ -dendroid, then $I(X)$ is a λ -dendroid.

Proof. Since a λ -dendroid is by definition a hereditarily unicoherent, hereditarily decomposable continuum, 3.7 follows directly from 3.4 and 3.6.

3.8. *Theorem.* If X is a compactum and each component is atriodic, then $I(X)$ is atriodic.

3.9. *Theorem.* If X is a compactum and each component is either a point or an arc-like continuum, then $I(X)$ is an arc-like continuum.

Proof. Notice that each component of X is hereditarily unicoherent and atriodic. Thus by 3.6 and 3.8, $I(X)$ is hereditarily unicoherent and atriodic. If a nondegenerate subcontinuum Y of $I(X)$ is indecomposable, then Y is a subcontinuum of a component of X . In this case Y is arc-like. Thus $I(X)$ is hereditarily unicoherent, atriodic, and every nontrivial

indecomposable subcontinuum of $I(X)$ is arc-like. By Theorem 2 of [9], $I(X)$ is arc-like.

Remarks. The reader should compare 3.9 with Theorem 2.1 of [16], and Theorem 11 of [5]. Theorem 3.9 answers the question posed by A. Lelek in a conversation with the author, "Can every compactum whose components are arc-like be imbedded in an arc-like continuum?"

3.10. *Corollary.* *If X is a compactum and each nondegenerate component of X is arc-like, then X is planar.*

Proof. Bing [3] has proved that every arc-like continuum is planar. This and 3.9 yield the corollary.

3.11. *Theorem.* *If X is a compactum and each component is either a point or a tree-like continuum, then $I(X)$ is a tree-like continuum.*

Proof. Again we note that each component of X is hereditarily unicoherent. Thus $I(X)$ is hereditarily unicoherent by 3.6. If Y is a nontrivial indecomposable subcontinuum of $I(X)$, then Y is a subcontinuum of a component of X . Since every subcontinuum of a tree-like continuum is tree-like, Y is tree-like. Hence $I(X)$ is hereditarily unicoherent and every nontrivial indecomposable subcontinuum of $I(X)$ is tree-like. By Theorem 1 of [6], $I(X)$ is tree-like.

3.12. *Theorem.* *If X is a compactum and each component is of dimension at most k ($k > 0$), then $I(X)$ is of dimension at most k .*

Proof. The continuum $I(X)$ is the union of X and $I(X) \setminus X$. The set $I(X) \setminus X$ consists of a countable number of components each of which is homeomorphic to either an open interval or a half open interval. Thus $I(X) \setminus X$ is the countable union of arcs. We note that X is of dimension at most k ($k > 0$). From

Theorem III 2 of [10], we get our theorem.

3.13. *Theorem.* If X is a compactum and each component is acyclic, then $I(X)$ is acyclic.

Proof. Let g be a mapping of $I(X)$ into the circle. The mapping is either essential or it is not. If the mapping is not essential, then it is inessential and in this case is homotopic to a constant mapping.

If g is an essential mapping of $I(X)$ into the circle S^1 , then there exists in $I(X)$ a continuum K with the property that $g|K$ is not homotopic to a constant mapping, but every proper subcontinuum K' of K is such that $g|K'$ is homotopic to a constant mapping. Furthermore, K is discoherent ([4], p. 216). The continuum K is *discoherent* if the complement of each subcontinuum of K is connected (refer to [15], p. 163). If $f(K)$ is not a point, then K is unicoherent. This we have proved earlier. Thus K must be a subcontinuum of one of the components of X . Since $g|K$ is not homotopic to a constant mapping, g restricted to the component N of X containing K is not homotopic to a constant mapping. It follows that N is not acyclic. This contradicts the fact that N is acyclic.

Since g is not an essential mapping, g is an inessential mapping. Also since g was arbitrary, we have that $I(X)$ is acyclic.

3.14. *Theorem.* If X is a compactum where each component is rational, and X contains at most a countable number of non-degenerate components, then $I(X)$ is rational.

3.15. *Theorem.* If X is a compactum where each component is Suslinian, and X contains at most a countable number of non-degenerate components, then $I(X)$ is Suslinian.

Remarks. There might be times when one would want to imbed

a compactum as described in the hypothesis of 3.15 in a locally connected Suslinian curve. Fitzpatrick and Lelek describe such an imbedding in [8]. It follows from their work that subcompacta of Suslinian continua are characterized by those properties described in the hypothesis of 3.15.

4. Characteristics of subcompacta of specific curves

Recall that a *curve* is a 1-dimensional continuum.

In this section we include some of the more immediate results which follow from our work in section 3.

4.1. *Theorem.* *A compactum X can be imbedded in a rational curve if and only if X contains at most a countable number of nontrivial components and each component is rational.*

This theorem follows directly from 3.14 and Theorem 5, page 285, in [15].

4.2. *Theorem.* *A compactum X can be imbedded in an acyclic curve if and only if the components of X are either degenerate or acyclic curves.*

Proof. If Y is an acyclic curve, then every subcontinuum of Y is an acyclic curve.

By 3.12 and 3.13, if every component of X is acyclic and at most 1-dimensional, then X can be imbedded in an acyclic 1-dimensional continuum.

4.3. *Theorem.* *A compactum X can be imbedded in an atriodic tree-like curve if and only if each nondegenerate component of X is atriodic and tree-like.*

The proof follows immediately from 3.8 and 3.11.

4.4. *Corollary.* *Either there exist nonplanar atriodic tree-like curves, or given any atriodic tree-like curve X, the plane contains uncountably many mutually disjoint homeomorphic*

copies of X.

Proof. Let X be an atriodic tree-like curve. Either X is nonplanar or it is not. If it is nonplanar, then the theorem is true. If X is planar, then $C \times X$, where C is the usual Cantor set, is a compactum and each component is atriodic and tree-like. Hence $C \times X$ can be imbedded in an atriodic tree-like curve Y by 4.3. Either Y is planar or it is not. If Y is planar, then the plane contains uncountably many mutually disjoint homeomorphic copies of X . If Y is not planar, then the first part of the theorem is true.

Remarks. This result relates two questions. Bing in [3] asks if given any atriodic tree-like planar curve X , does the plane contain uncountably many mutually disjoint homeomorphic copies of X ? Ingram [11] proved the existence of an atriodic tree-like curve in the plane which is not arc-like. Furthermore in [12], Ingram proved that uncountably many atriodic tree-like continua, none of which is arc-like, can be imbedded in the plane so that they are mutually exclusive. Ingram, at this conference, asked whether there exist nonplanar atriodic tree-like curves.

4.5. *Theorem.* A compactum X can be imbedded in a hereditarily decomposable continuum if and only if each nondegenerate component of X is hereditarily decomposable.

From 3.4 and the definition of hereditarily decomposable, the theorem easily follows.

4.6. *Theorem.* A compactum X can be imbedded in a hereditarily unicoherent curve if and only if each nondegenerate component of X is a hereditarily unicoherent curve.

Theorem 4.6 follows directly from 3.6 and the definition of hereditarily unicoherent.

4.7. *Theorem.* A compactum X can be imbedded in a λ -dendroid if and only if each nondegenerate component of X is a λ -dendroid.

This theorem follows directly from 3.7 and the definition of λ -dendroid.

Remarks. It is evident that we did not exhaust the possible results in this section. However, it is clear how one would proceed in order to get similar results.

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