Research Announcement:

ATRIODIC TREE-LIKE CONTINUA AND THE SPAN OF MAPPINGS

by

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1. Introduction

The problem of distinguishing among the tree-like continua those which are chainable has long been of interest. In particular, Bing [1] characterized the chainable continua among the hereditarily decomposable continua, and Fugate [6] further reduced the problem of determining chainability to determining chainability among indecomposable continua.

The author has studied atriodic tree-like continua which are not chainable showing that there are such continua [7] which are in some respects very much like chainable continua [8] and yet are quite far from being chainable [9] and [10]. The purpose of this paper is to raise some questions which seem to be pertinent to further study of such continua. Some of the questions raised here are questions which have been raised by others.

Throughout the paper all spaces are assumed to be metric spaces and all continua are assumed to be compact.

2. Plane embeddings

In view of a theorem of Bing [1] that every chainable continuum can be embedded in the plane it seemed reasonable to the author that one might try to obtain an atriodic tree-like continuum which is not chainable by constructing one which cannot be embedded in the plane. However, each such continuum that the author has found can be embedded in the plane [8] and [10].

Question 1. Is there an atriodic tree-like continuum

which cannot be embedded in the plane?

Bing [2] also characterized the circle-like continua which can be embedded in the plane.

Question 2. What characterizes the tree-like continua which can be embedded in the plane?

Question 3. (Bing [1, p. 656]). If M is an atriodic tree-like continuum in the plane, does there exist an uncountable collection of mutually exclusive continua in the plane each member of which is homeomorphic to M?

Michael Laidecker [11] has shown that Questions 1 and 3 are related. In particular he proves that if C is a compactum each component of which is an atriodic tree-like continuum then there exists an atriodic tree-like continuum which contains C.

3. Class W

Throughout this paper the term mapping means continuous function. The statement that the mapping f of a continuum X onto a continuum Y is *weakly confluent* (resp., *confluent*) means if K is a subcontinuum of Y then some (resp., each) component of $f^{-1}(K)$ is mapped by f onto K. If M is a continuum then the statement that M is in *Class W* [15] and [5, p. 21] means if f is a mapping of a continuum X onto M then f is weakly confluent.

David Read [17, Theorem 4, p. 236] showed that each chainable continuum is in Class W. Gary A. Feuerbacher [5, Theorem 7, p. 21] characterized the circle-like continua which are in Class W. H. Cook [3, Theorem 4, p. 243] has shown that each hereditarily indecomposable continuum is in Class W. The author has shown [9] that each continuum in the collection G of [8] is in Class W.

Question 4.   What characterizes the tree-like continua which are in Class W?

4. Weakly chainable continua

A. Lelek [12] and Lawrence Fearnley [4] independently characterized the continuous images of the pseudo-arc. A continuum is a continuous image of the pseudo-arc if and only if it is weakly chainable [12]. In [9] and [10] the author has shown the existence of atriodic tree-like continua which are not weakly chainable.

Question 5.   Is the example of [7] weakly chainable?

5. Span

Lelek [13] introduced span and proved that chainable continua have span zero. Span has proved crucial to the study of atriodic tree-like continua.

Suppose f is a mapping of a continuum X onto a continuum Y with metric d. The span of f, denoted by σf, is the least upper bound of $\{\varepsilon |$ there is a continuum Z in X×X such that $p_1(Z) = p_2(Z)$ and if (x,y) is in Z then $d(f(x),f(y)) \geq \varepsilon\}$ (Here, p_1 and p_2 denote the two projections of X×X onto X.) The surjective span of f, denoted by σ^*f, is defined similarly adding only that $p_1(Z) = X$. If M is a continuum, the span of M, denoted by σM, and the surjective span of M, denoted by σ^*M, are the span of the identity mapping on M and the surjective span of the identity mapping on M, respectively. Lelek [16] has shown that a simple triod can have surjective span smaller than span.

Question 6.   If M is a continuum, is $\sigma M = 0$ equivalent to $\sigma^*M = 0$? If f is a mapping, is $\sigma f = 0$ equivalent to $\sigma^*f = 0$?

Question 7.   (Lelek [14]). If M is a continuum and $\sigma M = 0$,

is M chainable?

   Question 8.   (Cook).   If M is a continuum, $\sigma M = 0$, and f
is a confluent mapping of M onto X, is $\sigma X = 0$?

   Question 9.   (Lelek [14]).   If M is a chainable continuum
and f is a confluent mapping of M onto X, is X chainable?
($\sigma X = 0$?)

   Theorem 4 of [7] gives a way to build, using inverse limits,
continua of positive span in terms of a condition on the bonding
maps.   It would be interesting to know conditions on the bonding
maps which would give a continuum of span zero.

   It would also be interesting to know how much analogy there
is between $\sigma f = 0$ (particularly for mappings to trees) and in-
essential mappings to the circle.

   The author has shown [10] that there are hereditarily in-
decomposable tree-like continua which are not chainable.

   Question 10.   (Cook).   Is there an homogeneous hereditarily
indecomposable tree-like continuum which is not chainable?

References

1.   R. H. Bing, *Snake-like continua*, Duke Math. J. 18 (1951),
     653-663.
2.   _____, *Embedding circle-like continua in the plane*,
     Canad. J. Math. 14 (1962), 113-128.
3.   H. Cook, *Continua which admit only the identity mapping onto
     non-degenerate subcontinua*, Fund. Math. 50 (1967), 241-249.
4.   L. Fearnley, *Characterization of the continuous images of
     the pseudo-arc*, Trans. Amer. Math. Soc. 111 (1964), 380-
     399.
5.   G. A. Feuerbacher, *Weakly chainable circle-like continua*,
     University of Houston dissertation.
6.   J. B. Fugate, *A characterization of chainable continua*,
     Canad. J. Math. 21 (1969), 383-393.
7.   W. T. Ingram, *An atriodic tree-like continuum with positive
     span*, Fund. Math. 77 (1972), 99-107.

8. _____, *An uncountable collection of mutually exclusive planar atriodic tree-like continua with positive span*, Fund. Math. 85 (1974), 73-78.

9. _____, *Concerning atriodic tree-like continua*, submitted to Fund. Math.

10. _____, *Hereditarily indecomposable tree-like continua*, submitted to Fund. Math.

11. M. Laidacker, *Imbeddings and universal compacta*, doctoral dissertation, University of Houston, 1976.

12. A. Lelek, *On weakly chainable continua*, Fund. Math. 50 (1962), 271-282.

13. _____, *Disjoint mappings and span of spaces*, Fund. Math. 55 (1964), 199-214.

14. _____, *Some problems concerning curves*, Colloq. Math. 23 (1971), 93-98.

15. _____, *Report on weakly confluent mappings*, preprint.

16. _____, *An example of a simple triod with surjective span smaller than span*, preprint.

17. D. R. Read, *Confluent and related mappings*, Colloq. Math. 29 (1974), 233-239.

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