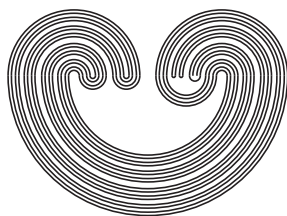

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LETTERS FROM THE R. L. MOORE PAPERS

by

LUCILLE WHYBURN

Topology Proceedings

Web: <http://topology.auburn.edu/tp/>

Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

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LETTERS FROM THE R.L. MOORE PAPERS

Lucille Whyburn

The R. L. Moore Collection in the Humanities Research Center of the University of Texas is a goldmine of letters, notebooks, handwritten corrections on reprints, underlined articles, and his own forthright statements.

I. Moore's First Original Contribution to Mathematics

From Austin, April 23, 1901, in pencil on some torn out pages of a notebook, speaking up in the first flush of discovery, R. L. Moore, age 19, wrote:

"Dear Papa:

"Hilbert (a German) perhaps one of the greatest, if not the very greatest, of the mathematicians of the present has formulated some assumptions from which, with possibly a slight addition, and the laws of pure logic and some elementary number concepts with the help of just about nothing else, it would seem a 'Euclidean' geometry can be built up. One of the assumptions seemed not as simple as some others of the group in which it appears and in a letter to Dr. Halsted (received by the latter about the 15th of April, I believe), Hilbert wrote to the effect that he believed that another assumption would do instead, it seems. It seems that he thought that the assumption in question would follow from the new assumption that he stated in the letter. I proved the new assumption itself from the other assumptions made (exclusive of the old one which was to be derived from the new) and also derived the old from the new, establishing the fact that this old

assumption in question was really not independent of the others but followed from them. I don't know that anyone (Hilbert not excepted) knew this before.

"Dr. Halsted has, I think, written to Hilbert concerning my discovery, but has not heard from him yet. I believe it takes quite a little while, about a month I believe, for a letter to go from here to Germany and an answer to come back. I think also that "notice" of the discovery will occur in the April or May number of the *Mathematical Monthly*.

"Hoping that all of the family are well and that I will see you in about a month and a half, I will close.

Your Affectionate Son,
Bob"

II. Moore, Kline, and Hilbert

J. R. Kline was Moore's most successful student from his University of Pennsylvania period and in Moore's early days at Texas they carried on a copious correspondence. It was on May 7, 1925 that Kline wrote Moore that the Guggenheim interests had given a sum of money which was to be used to aid professors on leave of absence to devote their full time to research abroad. He indicated he was applying and solicited Moore's support. Then proudly July 6 he wrote: "The Guggenheim people have granted me aid for research beginning February 1926 to February 1927. They made fifteen appointments and mine was the only one in mathematics."

Kline went to Göttingen, Germany, and he wrote Moore many delightful letters from there. On folded legal size stationery he wrote in longhand May 14, 1926, saying in part:

"Dear Moore,

"Your letter of April 22 arrived several days ago and I was very glad to hear from you. I waited to answer expecting to ask Alexandroff when I saw him the addresses of the people you mentioned. Alexandroff tells me that he and Urysohn became interested in Point Set Theory through the book of Hausdorff because when he was studying in Russia there was really no one who was interested in or competent to direct work in Analysis Situs."

"Alexandroff is very much interested in the things that are being done in America in Analysis Situs and values very highly your paper in the Transactions on the Foundations of Analysis Situs. Alexandroff seems to feel that it is mengen-theoretische and not combinatorial Analysis Situs which must prevail and says that the combinatorial is only a special case and can be regarded only as an indicator of how things can be done in the general case. He said that you had founded a wonderful Analysis Situs school in America which together with the Polish and Russian schools might push things very far."

"I drop in once in a while to Hilbert's lectures, not because I am particularly interested in the subject but I do want a chance to hear him lecture a few times. It is a most interesting performance. Along with the lecture there is carried on a soliloquy to the board by Hilbert; he will make a statement or prove a theorem and the steps will be interspersed with 'Ja, dass ist schön, wunderbar schön, etc.'. This however breaks out in full force when he gets stuck which happens at least once a lecture.

"Everyone in Göttingen seems to pride themselves on

Hilbert's excentricities (sic). Courant said about 20 years ago, he and Born, the physicist, were attending a Seminar of Hilbert's and as Hilbert was talking they noticed a little white place on the back of his trousers. He and Born bet as to whether it was a hole or chalk. However Hilbert soon answered that question by sitting down and as the white spot was not changed in form when he arose, they eliminated the chalk and decided it was a hole. Then they bet whether it would be there the next lecture, it was there and had grown to the size of a nickel. The next lecture it was the size of a quarter and everyone hesitated to tell Hilbert about it so it kept on until the people around the University were asking each other whether they had seen the hole in Hilbert's pants and comparing their estimates of the rate of growth of the hole. [The mathematics group had a picnic] and Hilbert rode his bicycle there which enlarged the hole exceedingly. Born decided it was really his duty to tell Hilbert, so after great hesitation he got Hilbert aside. 'Entschuldigen Sie Bitte, Ich weiss nicht wie es besten Ihnen zu sagen aber Herr Professor sie haben ein Loch in der Hosen.' Hilbert replied, 'Ja, Ja, das ist aber schon länger da und niemand hat es noch gesehen'."

On July 3, Kline wrote again, saying in part:

"Birkhoff came to town this morning and will be here for a week. I have not seen him as yet as he is staying with the Courants. He is here on a mission for the Rockefeller (sic) Foundation. I understand that the University here is asking a big sum of money (\$500,000 I have been told) from the

Rockerfeller Foundation."

"They had Hopf report on Birkhoff's mathematical work particularly the work that has appeared within the last five years in the Acta and his long memoir in the Transactions. Hilbert sat there for about an hour and a quarter and then started, 'Aber was sind die Haupt Gedanken? Was sind die grosse Sache dass daraus kommen? Geben sie nicht diese Kleinigkeiten aber was sind die grosse Satze?' Hopf replied that there was no big general theorem but there were a great number of separate theorems all proved with tremendous ingenuity. Then Hilbert began a long tirade about how the mathematicians always worked on small isolated problems and how the really big things were done by the physicists and the whole gist of his talk seemed to be that the mathematicians should have gotten Relativity instead of Einstein...Now Birkhoff is himself going to talk on Tuesday and everyone is waiting anxiously to see what will happen. Very likely Hilbert will not explode again."

"Hilbert is practically only interested in mathematical physics at the present time."

Dr. Moore was very admiring of Hilbert and later as President of the American Mathematical Society he proposed that Hilbert be brought as a speaker to the Summer meeting that was being planned in conjunction with the World's Fair in Chicago in 1938.

III. Moore, Hahn, and the Poles

R. L. Moore thought of Georg Cantor as the father of Point Set Theory and thought of Hans Hahn as having contributed some very fundamental results. Of course he was in

constant correspondence with the Poles, sending them problems and papers. Indeed some of the scratched out portions of his letters to the Poles are fascinating.

Thus when Gordon T. Whyburn became a Guggenheim Fellow in 1929-30, he and I went to Vienna. It was a happy choice. Vienna was a beautiful city not yet devastated by the war. Professor Hahn was a tall, handsome man, widely cultured, who introduced us to the Sunday afternoon concerts of the Vienna Symphony Orchestra, the local opera, and their theater live productions. He and Gordon developed a fine mathematical relationship. I think Gordon wrote a paper every month we were in Vienna. Karl Menger was there working on his dimension theory, Olga Taussky Todd was a graduate student, and the two of them introduced us to the fun of the Prater, Vienna's huge amusement park, and to the beautiful promenade along the Blue Danube where we strolled in the dusk of evening talking about the material and problems being presented in Hahn's Seminar. Gordon wrote back to Dr. Moore:

"We have just returned from a holiday trip. Immediately preceding the said Easter holidays, Knaster spent two weeks in Vienna; and during his visit here, we spent a very interesting time from the standpoint of point set theory. He is indeed a most likable person, and we enjoyed associating with him very much; and we are looking forward to further pleasant associations with him during our coming visit to Warsaw, for he has been so kind as to insist that we live with him while we are in Warsaw."

"Also during his stay here, I gave my lecture before the

mathematical Society in the German language. I spoke, as requested, on the cyclic elements of continuous curves, and it seemed to go off right well. I was rather pleased that some half hour's discussion followed my talk, with Hahn, Menger, and Knaster the principal participants.

"Knaster questioned me concerning your proof for the arcwise connectivity of connected and connected im kleinen G_δ -sets. It seems that Kuratowski had found a slight, easily remedied flaw in Menger's proof for this fact and also within another proof for it which has been published; and he asked Knaster to find out from me, if I knew, what your proof was like."

Moore answered promptly:

"Dear Dr. Whyburn:

"I am glad to have your letter of April 28. I was glad, too, to receive those of earlier dates and interested to hear of the progress of your investigations.

"You say that Knaster questioned you concerning my proof of the arcwise connectivity of connected and connected im kleinen G_δ -sets. In December, 1926, I presented to the American Mathematical Society a communication of which the abstract is printed on page 141 of vol. 33(1927) of the *Bulletin* of the American Mathematical Society.

"This is one of a number of theorems which I have stated but whose proofs I have refrained from publishing, preferring to publish them for the first time in my Colloquium book. Theorem 15 of my paper, 'On the foundations of plane analysis situs', states that if A and B are distinct points of a domain

M, there exists a simple continuous arc from A to B that lies wholly in M. Thus, in the above quoted abstract, I stated, in effect, that, in every space satisfying Axioms 1' and 2 and Theorem 4 of F.A., every connected domain is arcwise connected. But it is easy to see that if M is any connected and connected im kleinen G_δ -set lying in a space satisfying these axioms (and Theorem 4) then M, regarded as a space, itself satisfies these axioms (and Theorem 4 unless it is a single point). Hence every connected and connected im kleinen G_δ -set lying in such a space is arcwise connected and arcwise connected im kleinen. I can show that there exist spaces satisfying Axioms 1' and 2 and Theorem 4 of F.A. but which are not metric. Thus my theorem is more general than the theorem that in every complete metric space every connected and connected im kleinen G_δ -set is arcwise connected and arcwise connected im kleinen. I will enclose herewith a proof of the theorem, stated in my *Bulletin* abstract of 1927, to the effect that, in every space satisfying Axioms 1' and 2 and Theorem 4 of F.A., every connected domain is arcwise connected. This proposition remains true if Axiom 1' is replaced by any one of a number of other modifications of Axiom 1 which I have formulated, including those mentioned in my Boulder lectures. Will you please give the enclosed sheets to Doctor Knaster and also, if you wish, show him this letter."

"Kindly remember me to Mrs. Whyburn and Drs. Knaster, Kuratowski, Mazurkiewicz, Sierpinski, and Zarankiewicz."

Shortly then Gordon and I set off to meet these famous

mathematicians who were friends of Moore. Their exciting, stimulating mathematical atmosphere and cordiality surpassed our fondest dreams. Gordon wrote to Dr. Moore:

"Just a line as we are just back from Poland and I am very busy on a paper which Kuratowski and I are writing together on cyclic elements.

"We had a great time in Poland and enjoyed our visit there much. We spent a week in Warsaw and ten days in Lemberg.

"[We were so busy being entertained and were received by everyone], even including the President of Poland in Warsaw (Sierpinski and his wife got us into that), and I was kept so busy preparing and giving lectures in foreign languages (foreign both to myself and to the Poles) that it was next to impossible to do any work."

IV. R. L. Moore, Veblen, and E.H. Moore

R. L. Moore was at the height of his glory in the late nineteen twenties, working right on the frontier of mathematical research, rising in reputation both in America and Europe. It was in the summer of 1929 that he was able to present a set of Colloquium Lectures on his beloved Continuity Analysis Situs, Point Set Theory. Five years before in 1924 when Oswald Veblen was President of the AMS, Veblen had set up an Analysis Situs Committee to prepare a progress report and had invited R. L. to join it. Of course Dr. Moore would have had to spend several days on the train, traveling at his own expense, and missing his classes. He refused and suggested J. R. Kline. Veblen wrote back saying that he was sorry that Moore did not feel that he could serve on the Analysis Situs

Committee. After talking things over with Alexander, he said, he had decided not to say anything to Kline at present. Moore wrote back immediately in defense of his student and bidding for his own place in the sun. Veblen's letter was dated March 26, 1924; Moore's letter was dated April 2, 1924.

"Dear Veblen:

"I am sorry that you seem doubtful as to whether to add Kline to that committee. It seems to me that if he is omitted there will be a large field that will probably not be adequately treated. Quite a body of results has been established recently relating to continuous curves, im kleinen properties, prime parts of continua, etc. This branch of Analysis Situs has never been treated (unless to a slight extent) in a colloquium (your colloquium lectures dealt largely with combinatorial Analysis Situs) and I hope it will not also be neglected in this report. Kline is both able and careful and I feel that he would make a very satisfactory report in point-set theoretic Analysis Situs. Try him and see!"

It was E. H. Moore not Veblen who had appreciated R. L.'s 1916 paper "On the foundations of plane analysis situs" and who had fostered its publication in the *Transactions*. It was E. H. Moore who had written to E. R. Hedrick, managing editor of the *Bulletin* of the AMS, and told him that it would be safe to publish anything R. L. Moore submitted to him without bothering to have it refereed, and who sponsored R. L.'s papers in the *Proceedings* of the National Academy. R. L. Moore was not quite sure whether Veblen would have preferred that he not branch out on his own whereas he felt that E. H.

Moore understood the significance of his work and was more sympathetic to his point of view. Nevertheless Oswald Veblen was proud of his student R. L. Moore. I believe it was from Veblen's paper of 1904 that Dr. Moore became interested in the Heine-Borel Theorem that he used in his Calculus course when I took it. Also I think Veblen and Lennes's book *Introduction to Infinitesimal Analysis, Functions of One Real Variable* was the basis on which he built P.M. 24. He knew Veblen's early work very, very well.

V. Moore and Solomon Lefschetz

Maybe Moore felt that Veblen would have preferred that he work along the lines that Lefschetz was pursuing. In any case there was always a friendly rivalry between them. Lefschetz won sometimes. In 1928 Alexandroff wanted to come to America. Lefschetz was able to find him a stipend and Kline wrote sadly:

"He [Alexandroff] seems to have gone over to the Combinatorial Analysis Situs group almost entirely and is now trying to show that all that can be done in Continuity Analysis Situs can also be done by combinatorial methods (and I imagine he would say in the proper circles that it could be done easier) but if you will look at the end of his paper on Simplex Approximation there seems to me to be an admission that the combinatorial methods do not suffice for all problems which even he conceives."

Sometimes Moore won:

"March 7, 1925

"Dear Kline:

"I received, about five minutes ago, a letter from Lefschetz in which he wrote:

'I saw Kline and Gehman at the New York meeting last Saturday. Gehman is applying for a National Research Fellowship and asks to be sent to Texas. I strongly urged him to come to Princeton for a year first and get all he could from the local Analysis Situs before going to you. I think it would be a very excellent thing for both gangs, the local and yours. My suggestion was that the year after he should go down to Texas and thus establish the bridge. What do you think of it?'

"As to what I think of it--it doesn't sound sincere to me. If Gehman wants to go into the Princeton line of Analysis Situs--let him go with his eyes open. But don't let him go with his eyes half-shut, led by some pretence that in that way he will be better prepared to come down here. He has started a definite line of work. If he wants to continue that line let him do it. If he doesn't, let him do that. But let's not do the second under the guise of thereby doing the first! What is Gehman's reaction?"

Gehman chose Texas.

VI. Moore and E. B. Wilson

R. L. Moore loved intellectual kidding and once when E. B. Wilson was Managing Editor of the National Academy of Sciences Proceedings, Moore got a bill for some reprints he had ordered. He paid the bill. Later he received another bill for these reprints, the second bill being for a larger amount. An interchange of letters followed.

In reply to a letter of protest from Moore, Wilson wrote back to say there was a mix-up in billing because of a change in routine. Then tauntingly, he says that the *Proceedings* never adds interest to its bills and besides that a mathematician ought to have a keen sense of logic and know that \$1.60 could not be interest on \$8.40. He says that the editors are always glad to have Moore's papers because they are always high grade, however, he thinks Moore ought to have used a little sense in his letter. He goes on to explain that he is not making these remarks in his official capacity as Managing Editor of the *Proceedings* because he deems it to be his duty in that capacity to avoid controversy with authors and explain patiently about business matters, but rather he is speaking as a mathematician because, lamentably, there is a broad feeling among the populace that mathematicians do not have much sense which, he feels, all mathematicians should combat. R. L. Moore replied:

"Dear Professor Wilson:

"Your interesting epistle of October 2 has been received. Were it not for the way in which you have sweetened (?) your otherwise rather strong remarks I might be inclined to say I am a native Texan (as well as a mathematician) and that a real Texan is never satisfied with the rendering of a verbal response to such an attack. Personally I am not very much disturbed by such considerations as to whether people at large think that mathematicians are devoid of sense, but I myself know of some mathematicians who are devoid (or at least seem to be partially devoid) of a sense of humor. If there is a

broadly disseminated impression that mathematicians are devoid of sense I wonder whether 'sense' is not here used in the sense of 'business sense' and whether perhaps the responsibility for the dissemination of this impression does not rest largely on those mathematicians who, in attending to affairs of a business nature, fail to keep their records straight--who for example do such things as sending out the same bill twice, the second time some months after payment has been received--or rather ~~send~~ send it out once for one amount and later for a larger amount, and then become nettled if the recipient of the bills in question sarcastically inquires whether interest (usurious or otherwise) has been added.

"I am tempted to pursue these reflections further but will desist.

"I want to thank you for your good wishes. It is a source of some satisfaction to me that you at least think I have some mathematical sense."

VII. Moore and Pedagogy

R. L. Moore was a great teacher, loved, honored, and respected by his students. His normal teaching load throughout his long career was fifteen hours a week. He did not look upon teaching as a burden but rather as an opportunity. Indeed, the best way to further his branch of Analysis Situs from Moore's point of view was to find and develop the talent around him. In a letter to Veblen in 1928 he speaks frankly about this, saying: "If a man is vitally interested in training investigators and in trying to discover future investigators in undergraduate classes he may prefer to carry

on a certain amount of teaching both graduate and undergraduate, and his teaching may act as a stimulus to his researches."

To be a "teacher of teaching" was not his interest. He uses the full force of his delight in intellectual humor to make that point in his reply to an invitation to receive an honorary degree:

"I want to thank you for the invitation extended to me in your letter of March 7. But I don't feel that the degree of Doctor of Pedagogy would be an appropriate one for me.

"It is quite true that I am very much interested, and very much engaged, in a certain type of teaching but I don't think that fact would justify my being called a Doctor of Pedagogy. That title might suggest I teach teaching. And isn't there a great difference between teaching teaching and teaching? Indeed is it not true that most professional teachers of teaching are not very good teachers (even of teaching)?..If some students in, say a class in mathematics, taught by Professor A like his method of teaching and, later on, employ his methods, or modifications of them, in their own teaching, I don't think that would be enough to convict Professor A of being a professional teacher of teaching. I leave here aside the question of whether or not he may be better than a professional! But if he is not a professional teacher of teaching then does he deserve to have conferred upon him the Degree of Doctor of Pedagogy?

"In view of these and other considerations I don't think I should accept your invitation. However I do appreciate it."

Being a Moore student was hard work but it was not dull, not burdensome because Moore was such fun. Moreover he was highly motivated toward and skillful at arousing student ambition and self-development. John Worrell after taking an M.D. in medicine decided to return to Austin and obtain a Ph.D. under Moore. He may have expressed what many students feel when he wrote:

"There is nothing I have experienced since Austin which compares with the adventure in ideas through which you have led me. It is not mathematics alone but mathematics as you have shown it to me that keeps me in such unrest."

LISTING OF NAMES

Alexander, J. W. (1888-1971)	Knaster, Bronislaw (1893)
Alexandroff, Paul S. (1896)	Kuratowski, Kazimierz (1896)
Birkhoff, G. D. (1894-1944)	Lefschetz, Solomon (1888-1945)
Born, Max (1882-1970)	Mazurkiewicz, Stefan (1888-1945)
Cantor, Georg (1845-1918)	Menger, Karl (1902)
Courant, Richard (1888-1972)	Moore, E. H. (1862-1932)
Gehman, Harry M. (1899)	Moore, Robert Lee (1882-1974)
Hahn, Hans (1879-1934)	Sierpinski, Wacław (1888-1969)
Halsted, George Bruce (1853-1922)	Taussky, Olga (1906)
Hausdorff, Felix (1868-1942)	Veblen, Oswald (1880-1960)
Hedrick, E. R. (1876-1943)	Whyburn, Gordon Thomas (1904-1969)
Hilbert, David (1862-1929)	Wilson, E. B. (1879-1959)
Hopf, Heinz (1894-1971)	Zarankiewicz, Kazimierz (1902-1959)
Kline, John Robert (1891-1955)	Urysohn, Paul S. (1898-1924)

The University of Texas

Austin, Texas 78712