

---

# TOPOLOGY PROCEEDINGS



Volume 3, 1978

Pages 293–294

---

<http://topology.auburn.edu/tp/>

**Research Announcement:**  
CONTRACTIBLE DENDROIDS AND  
EMBEDDINGS IN THE PLANE

by

LEX G. OVERSTEEGEN

---

## Topology Proceedings

**Web:** <http://topology.auburn.edu/tp/>

**Mail:** Topology Proceedings  
Department of Mathematics & Statistics  
Auburn University, Alabama 36849, USA

**E-mail:** [topolog@auburn.edu](mailto:topolog@auburn.edu)

**ISSN:** 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

## CONTRACTIBLE DENDROIDS AND EMBEDDINGS IN THE PLANE

**Lex G. Oversteegen**

By a *continuum* we mean a compact connected metric space. A *dendroid* is a hereditarily unicoherent and arc-wise connected continuum. By a *fan* we understand a dendroid which contains at most one branch-point and we call this point the *vertex* of the fan. The following theorems have been proved:

*Theorem 1.* Let  $X$  be a fan which is locally connected at its vertex, then  $X$  is embeddable in the plane.

*Theorem 2.* Let  $X$  be a contractible fan, then  $X$  is locally connected at its vertex.

*Corollary 3.* Let  $X$  be a contractible fan, then  $X$  is embeddable in the plane.

We say that a space  $X$  is *monotone contractible* provided there exists a contraction  $H: X \times I \rightarrow X$  such that  $H|X \times \{t\}$  is monotone for each  $t \in I$ . Using some results of (4) we provide an internal characterization of contractibility for fans and prove:

*Theorem 4.* Let  $X$  be a fan, then the following are equivalent:

- i.  $X$  is contractible.
- ii.  $X$  is monotone contractible.

**Remarks.** Theorem 1 gives a solution to problem 1015 of

(3) and Corollary 3 answers a question (problem 786) of (2). We also answer a question raised in (1) and show that none of the above can be generalized to the class of dendroids. A complete version of this paper will appear elsewhere.

Acknowledgments. This research was supported by a Graduate Fellowship from Wayne State University. The author wishes to thank Professor A. Lelek for his advice.

### References

- (1) D. P. Bellamy and J. J. Charatonik, *The set function  $T$  and contractibility of continua*, Bull. Acad. Polon. Sci. 25 (1977), 47-49.
- (2) J. J. Charatonik and C. A. Eberhart, *On contractible dendroids*, Colloq. Math. 25 (1972), 89-98.
- (3) J. J. Charatonik and Z. Rudy, *Remarks on non-planable dendroids*, Fund. Math. (to appear).
- (4) B. G. Graham, *On contractible fans*, Doctoral dissertation, University of California, Riverside, California (1977).

Tulane University

New Orleans, Louisiana 70118