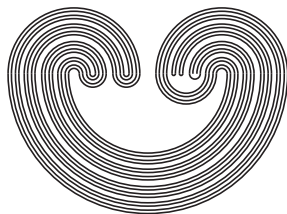

TOPOLOGY PROCEEDINGS



Volume 3, 1978

Pages 523–525

<http://topology.auburn.edu/tp/>

Research Announcement:
AN APPLICATION OF TREES TO
TOPOLOGY

by

Scott W. Williams

Topology Proceedings

Web: <http://topology.auburn.edu/tp/>

Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

AN APPLICATION OF TREES TO TOPOLOGY

Scott W. Williams

A *tree* is a POSET in which the set of predecessors of any element is well-ordered.

The *Gleason space* [C.N.], $\mathcal{G}(X)$, of a regular space X is the Stone space of the regular-closed set algebra $\mathcal{R}(X)$ of X . Two spaces X and Y are *\mathcal{G} -absolute* iff $\mathcal{G}(X)$ and $\mathcal{G}(Y)$ are homeomorphic. Note if X and Y are compact, *\mathcal{G} -absoluteness* is just *co-absoluteness* [Po; Wo].

Theorem 1. X is *\mathcal{G} -absolute* with a linearly ordered space if, and only if, $\mathcal{R}(X) - \{X\}$ contains a cofinal tree.

Application 1. A Moore space X is *\mathcal{G} -absolute* with a linearly ordered space if, and only if, X has a dense metrizable subspace.

Remark 1. A recent result in [Wh] suggests that "Moore" in the preceding might be replaced by "1st countable"; however, if X is a Souslin line, then X provides the counterexample. Further, $X \times [0,1]$ is not *\mathcal{G} -absolute* with any linearly ordered space.

A POSET P is *κ -closed* [B], for a cardinal κ , iff every well-ordered increasing sequence in P , of length κ , is bounded above. P is *separative* [Je] iff given $p \not\leq q \exists r \geq q \exists s$
 $s \geq p$ and $s \not\geq r$.

Theorem 2. A *κ -closed separative POSET* of cardinal $\leq 2^\kappa$ has a cofinal tree.

Application 2. If D is a locally compact non-compact metric space, then $\beta D - D$ is co-absolute with a linearly ordered space having a dense set of P -points.

Remark 2. S. Broverman has verbally communicated a short proof to the author that if D is an infinite discrete space of cardinal $\leq 2^\omega$, then $\beta D - D$ and $\beta N - N$ are co-absolute. We note that the same technique shows that if D is a dense-in-itself locally compact non-compact metric space of density $\leq 2^\omega$, then $\beta D - D$ and $\beta R - R$ are co-absolute. (N is the space of natural numbers and R is the reals.)

We consider two statements:

- (#) \exists precisely one (up to an isomorphism) complete atomless Boolean algebra B with a dense set of cardinal 2^ω and ω -closed.
- (*) If D is a locally compact non-compact metric space of density $\leq 2^\omega$, then $\beta D - D$ is co-absolute with $\beta N - N$.

The following theorem can be proved similarly to [G.J., 13.13]; however, our proof via trees appears new.

Theorem . $CH \Rightarrow \#$ and $2^\omega = 2^{\omega_1} \Rightarrow \neg \#$.

Application 3. $\# \Rightarrow (*)$ and $(*)$ is consistent with $\neg \#$.

Remark 3. If \mathcal{M} is a model of ZFC obtained from a model of CH by adding ω_2 Cohen reals iteratively [B], then $(*)$ is true in \mathcal{M} .

Corollary [Wo]. (CH) If D is a dense-in-itself locally compact non-compact metric space of density $\leq 2^\omega$, then $\beta D - D$ is co-absolute with $\beta N - N$.

Questions. (1) Does there exist a "real" example of a compact 1st countable space not \mathcal{G} -absolute with any linearly ordered space?

(2) Does $\# \Rightarrow CH$?

(3) Is $\beta R - R$ co-absolute with $\beta N - N$?

(4) Is $(\beta R - R)^2$ (respectively, $(\beta N - N)^2$) co-absolute with $\beta R - R$ ($\beta N - N$)?

References

- [B] J. P. Burgess, *Forcing, Handbook of mathematical logic*, North-Holland, 1977, 403-452.
- [C.N.] W. W. Comfort and S. Negrepointis, *The theory of ultrafilters*, Grand. Math. Wiss. Bd. 211, Springer-Verlag, 1974.
- [G.J.] L. Gillman and M. Jerison, *Rings of continuous functions*, Springer-Verlag, 1976.
- [Je] T. Jech, *Lectures in set theory*, Lect. Notes in Math. 217, Springer-Verlag, 1971.
- [Po] V. Ponomarev, *On the absolute of a topological space*, Soviet Math. 4 (1963), 299-302.
- [Wh] H. E. White, *First countable spaces having special pseudo-bases*, Can. Math. Bull. 21 (1978), 103-112.
- [Wo] R. G. Woods, *Co-absolutes of remainders of Stone-Čech compactifications*, Pac. J. Math 37 (1971), 545-560.

State University of New York at Buffalo

Buffalo, New York 14214