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Research Announcement:

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REFINABLE MAPS ON ANR'S

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In [F,R], Ford and Rogers define a map F: X ↠ Y to be *refinable* if for each $\varepsilon > 0$ there is an ε-map from X onto Y whose distance from F is less than ε. They ask if a refinable map defined on a compact ANR must map onto an ANR. One reason this question is of interest is that Kozlowski has shown that a refinable map onto an ANR is a cell-like map [K,R] and hence by a result of Ferry [F] a refinable map defined on an n-manifold, $n > 4$, is a near homeomorphism if the image is an ANR.

The following is a brief outline of the main results from [F,K], "Refinable Maps on ANR's."

I. Geometric Results

(1) If F: X ↠ Y is refinable, and X is a finite-dimensional compact ANR, then Y is an ANR if Y is LC^1 at each point. [Note: This result for infinite-dimensional ANR's will appear in a forthcoming paper by Kozlowski and Torunczyk.]

(2) If F: X ↠ Y is refinable and X is a finite dimensional compact ANR, then Y is LC^1 at each point (and hence an ANR) if either

(a) $F^{-1}(y)$ is nearly 1-movable for all $y \in Y$,

or (b) $F^{-1}(y)$ is locally connected for all $y \in Y$,

or (c) $F^{-1}(y)$ is approximately 1-connected for all $y \in Y$,

or (d) F has a monotone ε-refinement for every $\varepsilon > 0$.

(3) If F: $S^3 \to S^3/A$ is refinable then F is a near-homeomorphism.

II. Algebraic Results

(1) If F: X $\twoheadrightarrow$ Y is refinable and X is a compact ANR then F induces an isomorphism on each Čech cohomology group of any compactum in the range. A similar result holds for homology.

Definition. A closed set A in X is "N-elementary in X with respect to the group G (or the R-module G)" if for any neighborhood U of A in X there is a neighborhood V of A such that the homomorphism $\check{H}^N(U;G) \to \check{H}^N(V;G)$ induced by the inclusion $V \to U$ has a finitely generated image.

(2) In a more general setting, suppose F: X $\twoheadrightarrow$ Y is a refinable map between compacta. Then,

(i) if B is a compactum in Y such that $F^{-1}(B)$ is N-elementary in X with respect to a group G, then F induces an isomorphism on the N^{th} Čech cohomology group of B, and

(ii) if every subcompactum of X is N-elementary in X with respect to Z for each N and B is any compactum in Y, then F induces an isomorphism of the Čech homology of B to the Čech homology of $F^{-1}B$.

(3) If F: X $\twoheadrightarrow$ Y is a refinable map between compacta then F induces an isomorphism on each Čech cohomology group of X that is finitely generated.

(4) If F: X $\twoheadrightarrow$ Y is refinable and X is an orientable n-gcm over R (see [W] or [M, S]), then Y is also an orientable n-gcm over R.

III. Questions

(1) Does there exist an ANR X containing a Case-Chamberlin set A such that $X \to X/A$ is refinable? (See [C,C] for the definition.)

(2) If $F: S^3 \to Y$ is refinable, must Y be an ANR?

(3) If $F: S^n \to S^n/A$ is refinable, then must A be cellular?

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