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## Research Announcement: REFINABLE MAPS ON ANR'S

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## REFINABLE MAPS ON ANR'S

Jo Ford and George Kozłowski

In [F,R], Ford and Rogers define a map  $F: X \rightarrow Y$  to be *refinable* if for each  $\epsilon > 0$  there is an  $\epsilon$ -map from  $X$  onto  $Y$  whose distance from  $F$  is less than  $\epsilon$ . They ask if a refinable map defined on a compact ANR must map onto an ANR. One reason this question is of interest is that Kozłowski has shown that a refinable map onto an ANR is a cell-like map [K,R] and hence by a result of Ferry [F] a refinable map defined on an  $n$ -manifold,  $n > 4$ , is a near homeomorphism if the image is an ANR.

The following is a brief outline of the main results from [F,K], "Refinable Maps on ANR's."

### I. Geometric Results

(1) If  $F: X \rightarrow Y$  is refinable, and  $X$  is a finite-dimensional compact ANR, then  $Y$  is an ANR if  $Y$  is  $LC^1$  at each point. [Note: This result for infinite-dimensional ANR's will appear in a forthcoming paper by Kozłowski and Toruńczyk.]

(2) If  $F: X \rightarrow Y$  is refinable and  $X$  is a finite dimensional compact ANR, then  $Y$  is  $LC^1$  at each point (and hence an ANR) if either

- (a)  $F^{-1}(y)$  is nearly 1-movable for all  $y \in Y$ ,
- or
- (b)  $F^{-1}(y)$  is locally connected for all  $y \in Y$ ,
- or
- (c)  $F^{-1}(y)$  is approximately 1-connected for all  $y \in Y$ ,
- or
- (d)  $F$  has a monotone  $\epsilon$ -refinement for every  $\epsilon > 0$ .

(3) If  $F: S^3 \rightarrow S^3/A$  is refinable then  $F$  is a near-homeomorphism.

## II. Algebraic Results

(1) If  $F: X \rightarrow Y$  is refinable and  $X$  is a compact ANR then  $F$  induces an isomorphism on each Čech cohomology group of any compactum in the range. A similar result holds for homology.

*Definition.* A closed set  $A$  in  $X$  is "N-elementary in  $X$  with respect to the group  $G$  (or the  $R$ -module  $G$ )" if for any neighborhood  $U$  of  $A$  in  $X$  there is a neighborhood  $V$  of  $A$  such that the homomorphism  $\check{H}^N(U;G) \rightarrow \check{H}^N(V;G)$  induced by the inclusion  $V \rightarrow U$  has a finitely generated image.

(2) In a more general setting, suppose  $F: X \rightarrow Y$  is a refinable map between compacta. Then,

(i) if  $B$  is a compactum in  $Y$  such that  $F^{-1}(B)$  is N-elementary in  $X$  with respect to a group  $G$ , then  $F$  induces an isomorphism on the  $N^{\text{th}}$  Čech cohomology group of  $B$ , and

(ii) if every subcompactum of  $X$  is N-elementary in  $X$  with respect to  $Z$  for each  $N$  and  $B$  is any compactum in  $Y$ , then  $F$  induces an isomorphism of the Čech homology of  $B$  to the Čech homology of  $F^{-1}B$ .

(3) If  $F: X \rightarrow Y$  is a refinable map between compacta then  $F$  induces an isomorphism on each Čech cohomology group of  $X$  that is finitely generated.

(4) If  $F: X \rightarrow Y$  is refinable and  $X$  is an orientable  $n$ -gcm over  $R$  (see [W] or [M,S]), then  $Y$  is also an orientable  $n$ -gcm over  $R$ .

### III. Questions

(1) Does there exist an ANR  $X$  containing a Case-Chamberlin set  $A$  such that  $X \rightarrow X/A$  is refinable? (See [C,C] for the definition.)

(2) If  $F: S^3 \rightarrow Y$  is refinable, must  $Y$  be an ANR?

(3) If  $F: S^n \rightarrow S^n/A$  is refinable, then must  $A$  be cellular?

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