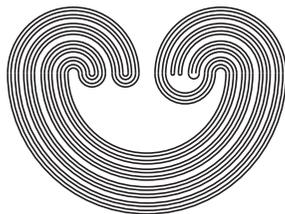

TOPOLOGY PROCEEDINGS



Volume 6, 1981

Pages 201–202

<http://topology.auburn.edu/tp/>

Research Announcement:

COMMON FIXED POINT THEOREMS

by

S. A. NAIMPALLY, K. L. SINGH, AND J. D. M. WHITFIELD

Topology Proceedings

Web: <http://topology.auburn.edu/tp/>

Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: 0146-4124

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COMMON FIXED POINT THEOREMS

S.A. Nainpally, K.L. Singh, and

J.H.M. Whitfield

Definition. (Takahashi) Let X be a metric space and I be closed unit interval. A mapping $W: X \times X \times I \rightarrow X$ is said to be a *convex structure* on X if for all $x, y \in X$ and $\lambda \in I$ the following condition is satisfied

$$d(u, W(x, y; \lambda)) \leq \lambda d(u, x) + (1-\lambda)d(u, y)$$

for all u in X . A metric space with a convex structure is called a *convex metric space*.

Theorem 1. Let K be a nonempty compact convex subset of a convex metric space X . If S is a left reversible semigroup of nonexpansive mappings of K into itself, then K contains a common fixed point of S .

Theorem 2. Let X be a compact metric space and $G: X \rightarrow X$ be a linearly ordered semigroup of mappings. Suppose G has diminishing orbital diameter and there exists $g \in G$ with $g \neq I$ such that

(i) G is continuous mapping with diminishing orbital diameter,

(ii) G is Archimedean at g .

Then G has a common fixed point.

Theorem 3. Let X be a convex metric space having property (C) and H be a closed convex subset of X . Let K be a bounded, closed convex subset of H with normal structure.

If $T: K \rightarrow H$ is nonexpansive and if $T: \partial_H K \rightarrow K$ ($\partial_H K$ is the relative boundary of $H \cap K$ in H), then T has a fixed point in K .

Also we prove a common fixed point theorem for commuting linearly ordered semigroup of nonexpansive mappings having convex diminishing orbital diameter.

Theorem 1 generalizes results of De Marr [1], Mitchell [4] and Takahashi [5,6]. Theorem 2 and 3 extend the results of Kirk [2], [3] respectively.

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Lakehead University
 Thunder Bay, Ontario
 Canada P7B 5E1