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RELATIONS BETWEEN HOMOLOGY AND HOMOTOPY PRO-GROUPS OF CONTINUA

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All basic notions of shape theory and pro-categories used in this paper can be found in [D-S].

In the recent paper [F] S. Ferry has characterized continua having the shape of an LC^n-space ($n \geq 0$, the case $n = 0$ is due to J Krasinkiewicz [K_2]) as those possessing the following property:

(*) for some point $x \in X$ the k-th homotopy pro-group pro-$\pi_k(X,x)$ is stable for $k \leq n$ and satisfies the Mittag-Leffler condition for $k = n + 1$.

Also the author proved in [D_3] that a continuum X is an LC^{n+1}-divisor ($n \geq 0$) iff X is nearly 1-movable and satisfies the following condition:

(**) the k-th homology pro-group pro-$H_k(X)$ is stable for $k \leq n$ and satisfies the Mittag-Leffler condition for $k = n + 1$.

The interplay between conditions (*) and (**) was investigated in [D_2]. Here is the main result of [D_2].

Theorem 1. *Let* $\underline{X} = (X_m, p_m^{m+1})$ *be an inverse sequence of pointed connected* CW *complexes and let* $n \geq 0$. *If* pro-$\pi_k(\underline{X})$ *is stable for* $k \leq n$ *and satisfies the Mittag-Leffler condition for* $k = n + 1$, *then* pro-$H_k(\underline{X})$ *is stable for* $k \leq n$ *and satisfies the Mittag-Leffler condition for*

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$k = n + 1$. *Also if* pro-$\pi_1(\underline{X})$ *is isomorphic to the trivial group, then the converse holds true.*

Condition (**) can be expressed with the use of Čech cohomology groups in the following way (see [D_1]):

Theorem 2. *Let* X *be a continuum. Then the following conditions are equivalent for* $n \geq 0$*:*

a. pro-$H_k(X)$ *is stable for* $k \leq n$ *and satisfies the Mittag-Leffler condition for* $k = n + 1$*,*

b. $\check{H}^k(X)$ *is finitely generated for* $k \leq n$*,* $\check{H}^{n+1}(X)/\text{Tor}\,\check{H}^{n+1}(X)$ *is a free Abelian group and* Tor $\check{H}^{n+1}(X)$ *is finite.*

Theorems 1 and 2 were used in [D_2] to give a simple proof of the following result due to Geoghegan and Lacher [G-L].

Theorem 3. *A* 1*-shape connected continuum* X *has the shape of a finite complex if the deformation dimension of* X *is finite and all Čech cohomology groups* $\check{H}^n(X)$ *are finitely generated.*

In this paper we provide some further applications of Theorem 1.

The main tool in proving Theorem 1 was (see [D_2])

Lemma 4. *Suppose*

$$p_n^{k-1,k}\colon G_n^k \to G_n^{k-1} \text{ and } q_{n,n+1}^k\colon G_{n+1}^k \to G_n^k$$

are homomorphisms of groups ($n \geq 1$ and $1 \leq k \leq 5$ is an integer) *such that*

$$p_n^{k-1,k} \cdot q_{n,n+1}^k = q_{n,n+1}^{k-1} \cdot p_{n+1}^{k-1,k}$$

for $n \geq 1$ *and* $2 \leq k \leq 5$, *each sequence* $\underline{G}_n = (G_n^k, p_n^{k-1,k})$ *is exact and* $\underline{G}^k = (G_n^k, q_{n,n+1}^k)$.

If G^1 *is stable and* G^i *satisfies the Mittag-Leffler condition for* $i = 2, 4$, *then* G^3 *satisfies the Mittag-Leffler condition.*

We need the following supplement to Lemma 4.

Lemma 5. *Under the hypotheses of Lemma* 4 *if* $\underline{G}^k$ *is stable for* $k = 1, 2, 4$ *and satisfies the Mittag-Leffler condition for* $k = 5$, *then* G^3 *is stable.*

Proof. By Lemma 4 the pro-group $\underline{G}^3$ satisfies the Mittag-Leffler condition.

Recall that the fact that a pro-group (A_n, p_n^m) satisfies the Mittag-Leffler condition (is stable) can be formulated in the following way:

there is an increasing sequence $n_1 < n_2 < \cdots$ of positive integers such that

$$p_{n_k}^{n_{k+1}} \Big| \operatorname{im}(p_{n_{k+1}}^{n_{k+2}}) : \operatorname{im}(p_{n_{k+1}}^{n_{k+2}}) \to \operatorname{im}(p_{n_k}^{n_{k+1}})$$

is an epimorphism (isomorphism).

Therefore without loss of generality we may assume that

$$q_{n,n+1}^k \Big| \operatorname{im}(q_{n+1,n+2}^k) : \operatorname{im}(q_{n+1,n+2}^k) \to \operatorname{im}(q_{n,n+1}^k)$$

is an isomorphism (epimorphism) for all $n \geq 1$ and $k = 1, 2, 4$ $(k = 3, 5)$. We are going to prove that

$$q_{n,n+1}^3 \Big| \operatorname{im}(q_{n+1,n+2}^3) : \operatorname{im}(q_{n+1,n+2}^3) \to \operatorname{im}(q_{n,n+1}^3)$$

is a monomorphism. So suppose

$$x_{n+1}^3 \in \operatorname{im}(q_{n+1,n+2}^3) \cap \ker(q_{n,n+1}^3).$$

Take $x^3_{n+3} \in \mathrm{im}(q^3_{n+3,n+4})$ such that

$$x^3_{n+1} = q^3_{n+1,n+3}(x^3_{n+3})$$

(by $q^k_{m,n}$ we denote the composition of corresponding $q^k_{p-1,p}$). Then $p^{2,3}_{n+3}(x^3_{n+3}) \in \mathrm{im}(q^2_{n+3,n+4})$ and $q^2_{n,n+3} \cdot p^{2,3}_{n+3}(x^3_{n+3}) = 1$. Therefore $p^{2,3}_{n+3}(x^3_{n+3}) = 1$ and there exists $x^4_{n+3} \in G^4_{n+3}$ with $x^3_{n+3} = p^{3,4}_{n+3}(x^4_{n+3})$. Since $p^{3,4}_n \cdot q^4_{n,n+3}(x^4_{n+3}) = 1$ there is $x^5_n \in G^5_n$ with $p^{4,5}_n(x^5_n) = q^4_{n,n+3}(x^4_{n+3})$. Take $x^5_{n+3} \in G^5_{n+3}$ with

$$q^5_{n-1,n+3}(x^5_{n+3}) = q^5_{n-1,n}(x^5_n).$$

Then

$$\begin{aligned} q^4_{n-1,n+2} \cdot p^{4,5}_{n+2} \cdot q^5_{n+2,n+3}(x^5_{n+3}) &= \\ = p^{4,5}_{n-1} \cdot q^5_{n-1,n+3}(x^5_{n+3}) &= p^{4,5}_{n-1} \cdot q^5_{n-1,n}(x^5_n) = \\ = q^4_{n-1,n} \cdot p^{4,5}_n(x^5_n) &= q^4_{n-1,n} \cdot q^4_{n,n+3}(x^4_{n+3}) = \\ = q^4_{n-1,n+2} \cdot q^4_{n+2,n+3}(x^4_{n+3}). \end{aligned}$$

Hence

$$q^4_{n+2,n+3}(x^4_{n+3}) = p^{4,5}_{n+2} \cdot q^5_{n+2,n+3}(x^5_{n+3})$$

and therefore

$$\begin{aligned} x^3_{n+1} = q^3_{n+1,n+3}(x^3_{n+3}) &= q^3_{n+1,n+3} \cdot p^{3,4}_{n+3}(x^4_{n+3}) = \\ &= p^{3,4}_{n+1} \cdot q^4_{n+1,n+3}(x^4_{n+3}) = \\ &= p^{3,4}_{n+1} \cdot q^4_{n+1,n+2} \cdot p^{4,5}_{n+2} \cdot q^5_{n+2,n+3}(x^5_{n+3}) = \\ &= q^3_{n+1,n+2} \cdot p^{3,4}_{n+2} \cdot p^{4,5}_{n+2} \cdot q^5_{n+2,n+3}(x^5_{n+3}) = 1. \end{aligned}$$

Thus $\underline{G}^3$ is stable.

The main result of this paper is the following:

Theorem 6. *Suppose* X_1 *and* S_2 *are continua such that* $X_1 \cap X_2 \neq \emptyset$ *and* $X_1 \cup X_2$ *have the shape of some* LC^m*-spaces*

($m \geq 1$). *If* X_1 *is pointed* 1-*movable and the natural homomorphism of first Čech homotopy groups*

$$\check{\pi}_1(X_1,x_0) \to \check{\pi}_1(X_1 \cup X_2,x_0)$$

is a monomorphism for some point $x_0 \in X_1 \cap X_2$, *then* X_1 *has the shape of an* LC^m-*space.*

Proof. Since pro-$\pi_1(X_1 \cup X_2,x_0)$ is stable, we get that $\check{\pi}_1(X_1 \cup X_2,x_0)$ is countable and therefore $\check{\pi}_1(X_1,x_0)$ is countable. Now Corollary 6.1.9 in [D-S] (p. 81) says that pro-$\pi_1(X_1,x_0)$ is stable. First consider the case where $X_1 \cap X_2$ is connected.

Take an inverse system (Z_n,q_n^{n+1}) of finite connected CW complexes such that for some connected subcomplexes $X_{1,n}, X_{0,n}$ and $X_{2,n}$ of Z_n there is

a. $X_{0,n} = X_{1,n} \cap X_{2,n}$ and $X_{1,n} \cup X_{2,n} = Z_n$,

b. $X_1 \cup X_2 = \varprojlim(Z_n,q_n^{n+1})$,

c. $X_1 = \varprojlim(X_{1,n},q_n^{n+1})$

d. $X_2 = \varprojlim(X_{2,n},q^{n+1})$

e. $X_0 = X_1 \cap X_2 = \varprojlim(X_{0,n},q_n^{n+1})$.

Without loss of generality we assume each q_n^{n+1} is cellular.

Moreover we may assume that each q_n^{n+1} induces isomorphisms of $\pi_1(X_{1,n+1},x_{n+1})$ onto $\pi_1(X_{1,n},x_n)$ (here $x_0 = (x_n)$) and of $\pi_1(X_{0,n+1},x_{n+1})$ onto $\pi_1(X_{0,n},x_n)$ (see [K_1], Theorem 3.1 on p. 151, or [F]). Also we assume that each $\pi_1(Z_n,x_n)$ contains $G = \check{\pi}_1(X_1 \cup X_2,x_0)$ such that $\pi_1(q_n^{n+1})$ maps G identically onto itself. We identify each group $\pi_1(X_{0,n},x_n)$ with $G_0 = \check{\pi}_1(X_0,x_0)$ and each $\pi_1(X_{1,n},x_n)$ with $G_1 = \check{\pi}_1(X_1,x_0)$ in such a way that q_n^{n+1} induces the identity on G_0 and G_1 when restricted to $X_{0,n+1}$ and $X_{1,n+1}$ respectively. For each n take the universal covering space $\tilde{Z}_n$ of Z_n and let $p_n\colon \tilde{Z}_n \to Z_n$ be the covering projection. Let $\overline{X}_{i,n} = p_n^{-1}(X_{i,n})$ for i = 0, 1, 2, and choose a base point $\tilde{x}_n \in p_n^{-1}(x_n)$ for each n. Take maps

$$\tilde{q}_n^{n+1}\colon (\tilde{Z}_{n+1},\tilde{x}_{n+1}) \to (\tilde{Z}_n,\tilde{x}_n)$$

such that

$$p_n \cdot \tilde{q}_n^{n+1} = q_n^{n+1} \cdot p_{n+1}.$$

Since each p_n induces isomorphisms of all k-homotopy groups for $k \geq 2$ and pro-$\pi_k(X_1 \cup X_2,x_0)$ is stable for $k \leq m$ and satisfies the Mittag-Leffler condition for k = m + 1, we infer that $(\pi_k(\tilde{Z}_n,\tilde{x}_n),\pi_k(\tilde{q}_n^{n+1}))$ is stable for $k \leq m$ and satisfies the Mittag-Leffler condition for k = m + 1. By Theorem 1 the pro-group $(H_k(\tilde{Z}_n),H_k(\tilde{q}_n^{n+1}))$ is stable for $k \leq m$ and satisfies the Mittag-Leffler condition for $k = m + 1$.

For each n consider the component $\hat{X}_{0,n}$ of $\overline{X}_{0,n}$ containing $\tilde{x}_n$.

Then $\overline{p}_n = p_n|\hat{X}_{0,n}\colon \hat{X}_{0,n} \to X_{0,n}$ is a covering projection such that $\mathrm{im}[\pi_1(\overline{p}_n)]$ is equal to the kernel of the natural homomorphism from $G_0 = \check{\pi}_1(X_0,x_0)$ to $G = \check{\pi}_1(X_1 \cup X_2,x_0)$. Therefore

$$\overline{q}_n^{n+1} = \tilde{q}_n^{n+1}\,(\hat{X}_{0,n+1},\tilde{x}_{n+1})\colon (\hat{X}_{0,n+1},\tilde{x}_{n+1}) \to (\hat{X}_{0,n},\tilde{x}_n)$$

induces isomorphisms on π_1. Similarly as before we get that $(H_k(\hat{X}_{0,n}),H_k(\overline{q}_n^{n+1}))$ is stable for $k \leq m$ and satisfies the Mittag-Leffler condition for k = m + 1. Now observe that $\tilde{q}_n^{n+1}$ induces a bijection between components of $\overline{X}_{0,n+1}(\overline{X}_{1,n+1})$ and $\overline{X}_{0,n}(\overline{X}_{1,n})$. Moreover each component of

$\overline{X}_{0,n}(\overline{X}_{1,n})$ is the image of $\hat{X}_{0,n}(\hat{X}_{1,n}$ which is the component of $\overline{X}_{1,n}$ containing $\tilde{x}_n)$ under an element of $g \in G$, where G is interpreted as a subgroup of $\pi_1(Z_n,x_n)$ which acts on $\tilde{Z}_n$ (see [Co], p. 12). Since the action of fundamental groups is functorial in the sense of 3.16 in [Co] (p. 12), one easily gets that $(H_k(\overline{X}_{0,n}),H_k(\tilde{q}_n^{n+1}|\overline{X}_{0,n+1}))$ is stable for $k \leq m$ and satisfies the Mittag-Leffler condition for $k = m + 1$.

Now for each n we have the functorial Mayer-Vietoris exact sequence

$$\to H_k(\overline{X}_{0,n}) \to H_k(\overline{X}_{1,n}) \oplus H_k(\overline{X}_{2,n}) \to H_k(\tilde{Z}_n)$$
$$\to H_{k-1}(\overline{X}_{0,n}) \to .$$

Applying Lemma to 4 and 5 we get that

$$(H_k(\overline{X}_{1,n}),H_k(\tilde{q}_n^{n+1}|\overline{X}_{1,n+1}))$$

is stable for $k \leq m$ and satisfies the Mittag-Leffler condition for $k = m + 1$. Since $(\overline{X}_{1,n},[\tilde{q}_n^{n+1}|\overline{X}_{1,n+1}])$ dominates $(\hat{X}_{1,n},[\tilde{q}_n^{n+1}|\hat{X}_{1,n+1}])$ in pro-homotopy, we infer that

$$(H_k(\hat{X}_{1,n}),H_k(\tilde{q}_n^{n+1}|\hat{X}_{1,n+1}))$$

is stable for $k \leq m$ and satisfies the Mittag-Leffler condition for $k = m + 1$. Recall that the natural homomorphism of G_1 to G is a monomorphism. Therefore each $\hat{X}_{1,n}$ is simply connected and Theorem 1 says that $(\pi_k(\hat{X}_{1,n},\tilde{x}_n), \pi_k(\tilde{q}_n^{n+1}|\hat{X}_{1,n+1}))$ is stable for $k \leq m$ and satisfies the Mittag-Leffler condition for $k = m + 1$. Consequently $(\pi_k(X_{1,n},x_n),\pi_k(q_n^{n+1})) = \text{pro-}\pi_k(X_1,x_0)$ is stable for $k \leq m$ and satisfies the Mittag-Leffler condition for $k = m + 1$ which completes the proof of Theorem 6 in case $X_1 \cap X_2$ is connected.

If $X_1 \cap X_2$ is not connected it has a finite number of components. Take an abstract arc L intersecting each component of $X_1 \cap X_2$ in exactly one point. Then $X_1' = X_1 \cup L$ and $X_2' = X_2 \cup L$ satisfy the hypotheses of Theorem 6 and $X_1' \cap X_2'$ is connected. Therefore $X_1 \cup L$ has the shape of an LC^n-space. By the main result of [K_2] (see also [D-S], p. 95) there is a sequence $Y_k \supset Y_{k+1} \supset \cdots$ of locally connected continua whose intersection is X_1 such that each Y_{k+1} is a strong deformation retract of Y_k. Then $Y_{k+1} \cup L$ is a strong deformation retract of $Y_k \cup L$ which implies that (X_1,x_0) is shape dominated by $(X_1 \cup L,x_0)$. Therefore X_1 has the shape of some LC^m-space. Thus the proof of Theorem 6 is concluded.

Corollary 7. *Suppose* X_1 *and* X_2 *are continua such that* $X_1 \cap X_2 \neq \emptyset$ *and* $X_1 \cup X_2$ *are pointed* ANSR*'s. If* X_1 *is a pointed* 1*-movable continuum of finite deformation dimension and the natural homomorphism* $\check{\pi}_1(X_1,x_0) \to \check{\pi}_1(X_1 \cup X_2,x_0)$ *is a monomorphism, then* X_1 *is a pointed* ANSR.

Proof. By Theorem 6 and Ferry's result [F] all homotopy groups of (X_1,x_0) are stable. By [E-G] (Theorem 5.1, see also [D-S], Theorem 9.22 on p. 114) X_1 is a pointed ANSR.

Corollary 8. *Suppose* X_1 *and* X_2 *are continua of finite deformation dimension such that* $X_1 \cap X_2$ *and* $X_1 \cup X_2$ *are pointed* ANSR*'s. If* $X_1 \cap X_2$ *is* 1*-shape connected, then* X_1 *and* X_2 *are pointed* ANSR*'s.*

Proof. First suppose $X_1 \cap X_2$ is connected and take $x_0 \in X_1 \cap X_2$ (the case $X_1 \cap X_2 = \emptyset$ is obvious). Since the projections $p: X_1 \cup X_2 \to (X_1 \cup X_2)|(X_1 \cap X_2)$ and $p|X_1: X_1 \to X_1|(X_1 \cap X_2)$ induce isomorphisms of first shape groups (see [D_4], Theorem 8.6 on p. 41) and clearly the inclusion $X_1|(X_1 \cap X_2) \to (X_1 \cup X_2)|X_1 \cap X_2$ induces monomorphism of first shape groups, we infer that the natural homomorphism from $\check{\pi}_1(X_1,x_0)$ to $\check{\pi}_1(X_1 \cup X_2,x_0)$ is a monomorphism. Then Corollary 7 implies that X_1 is a pointed ANSR.

If $X_1 \cap X_2$ is not connected we take an abstract arc L intersecting each component of $X_1 \cap X_2$ at exactly one point. If $X_1' = X_1 \cup L$ and $X_2' = X_2 \cup L$, then the hypotheses of Corollary 8 are satisfied and $X_1' \cap X_2'$ is connected. Therefore $X_1 \cup L$ is a pointed ANSR. Now Corollary 5.2 in [D-O] implies that X_1 is pointed 1-movable. In the same way as in the proof of Theorem 6 one gets that (X_1,x_0) is shape dominated by $(X_1 \cup L,x_0)$. Consequently X_1 is a pointed ANSR.

Remark. Both Corlllary 7 and 8 relate to the following problem posed by Borsuk [B]:

Is it true that X_1 and X_2 are ANSR's provided $X_1 \cup X_2$ and $X_1 \cap X_2$ are ANSR's?

A counterexample to this problem was provided by the author in [D_5] and independently by K. Kuperberg (unpublished).

Observe that using our methods one can prove the following

Theorem 9. *If compacta* X_1 *and* X_2 *have the shape of* LC^k *spaces* $(k \geq 0)$ *and* $X_1 \cap X_2$ *has the shape of an* LC^{k-1}*-space, then* $X_1 \cup X_2$ *has the shape of an* LC^k*-space.*

The proof of Theorem 9 is analogous to the proof of Theorem 6 and uses the following

Lemma 10. *Suppose* X_1*,* X_2 *and* $X_1 \cap X_2$ *are continua such that for some point* $x_0 \in X_1 \cap X_2$ *the pro-groups* pro-$\pi_1(X_1,x_0)$ *and* pro-$\pi_1(X_2,x_0)$ *are stable. If* $X_1 \cap X_2$ *is pointed* 1*-movable, then* pro-$\pi_1(X_1 \cup X_2,x_0)$ *is stable.*

Proof. Take an inverse sequence (Z_n,q_n^{n+1}) of finite connected CW complexes such that for some connected subcomplexes $X_{1,n}, X_{0,n}$ and $X_{2,n}$ of Z_n there is

a. $X_{0,n} = X_{1,n} \cap X_{2,n}$ and $Z_n = X_{1,n} \cup X_{2,n}$,

b. $X_1 \cup X_2 = \varprojlim(Z_n,q_n^{n+1})$,

c. $X_1 = \varprojlim(X_{1,n},q_n^{n+1})$

d. $X_2 = \varprojlim(X_{2,n},q_n^{n+1})$

e. $X_0 = X_1 \cap X_2 = \varprojlim(X_{0,n},q_n^{n+1})$.

Without loss of generality we may assume each q_n^{n+1} is cellular. Moreover we may assume that each q_n^{n+1} induces isomorphisms of $\pi_1(X_{1,n+1},x_{n+1})$ onto $\pi_1(X_{1,n},x_n)$ (here $x_0 = (x_n)$) and of $\pi_1(X_{2,n+1},x_{n+1})$ onto $\pi_1(X_{2,n},x_n)$, and an epimorphism of $\pi_1(X_{0,n+1},x_{n+1})$ onto $\pi_1(X_{0,n},x_n)$ (see [K], Theorem 3.1 on p. 151, or [F]).

Let us fix n for a moment and consider the kernel A of

$$\pi_1(q_n^{n+1})\colon \pi_1(X_{0,n+1},x_{n+1}) \to \pi_1(X_{0,n},x_n).$$

Notice that any loop α such that $[\alpha] \in A$ is contractible in both $X_{1,n+1}$ and $X_{2,n+1}$.

Therefore if we attach a family $\{D_j\}_{j\in J}$ of 2-discs to $X_{0,n+1}$ in order to kill A, the inclusions

$$X_{1,n+1} \to X_{1,n+1} \cup \bigcup_{j\in J} D_j \text{ and } X_{2,n+1} \to X_{2n+1} \cup \bigcup_{j\in J} D_j$$

induce isomorphisms of fundamental groups, and $X_{1,n+1} \cup X_{2,n+1}$ is a retract of $X_{1,n+1} \cup X_{2,n+1} \cup \bigcup_{j\in J} D_j$. Hence the inclusion $i_n: X_{1,n+1} \cup X_{2,n+1} \to X_{1,n+1} \cup X_{2,n+1} \cup \bigcup_{j\in J} D_j$ induces isomorphism of fundamental groups. Take any extension

$$\bar{q}_n^{n+1}: Z_{n+1} \cup \bigcup_{j\in J} D_j \to Z_n \text{ of } q_n^{n+1} \text{ with}$$

$$\bar{q}_n^{n+1}(\bigcup_{j\in J} D_j) \subset X_{0,n}.$$

Then

$$\bar{q}_n^{n+1}|X_{1,n+1} \cup \bigcup_{j\in J} D_j: X_{1,n+1} \cup \bigcup_{j\in J} D_j \to X_{1,n},$$

$$\bar{q}_n^{n+1}|X_{2,n+1} \cup \bigcup_{j\in J} D_j: X_{2,n+1} \cup \bigcup_{j\in J} D_j \to X_{2,n}$$

and

$$\bar{q}_n^{n+1}|X_{0,n+1} \cup \bigcup_{j\in J} D_j: X_{0,n+1} \cup \bigcup_{j\in J} D_j \to X_{0,n}$$

induce isomorphisms of fundamental groups and by van Kampen's Theorem $\bar{q}_n^{n+1}$ induces an isomorphism of fundamental groups.

Consequently $q_n^{n+1} = \bar{q}_n^{n+1} \cdot i_n$ induces an isomorphism of fundamental groups and the proof of Lemma 10 is concluded.

Remark. Theorem 9 can be derived from [Kod] under the weaker assumption that $X_1 \cap X_2$ has the shape of an LC^k-space.

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