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by

Douglas E. Cameron

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
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The first two decades of this century were years of turmoil and strife in Russia as political dissidents tried to secure a better life for themselves and their beloved Russia. Twice these efforts burst forth in the form of armed revolts: the first in 1905 ended in a pseudoconcilatory peace with Tsar Nicholas II promising reforms, but the respite was only temporary; the second in 1917 when violence again erupted and this time climaxed with the October 25th revolution[1] and the end of tsarist Russia.

But political unrest was not the only revolutionary spirit being evidenced in Russia. Within the walls of the educational system and especially at Moscow University reforms were occurring in the classrooms which, albeit peaceful, were to be extremely successful and beneficial to the Russian people and the people of the world in general. Through the early years of this century, scientific rhetoric in classrooms was strictly structured and its main emphasis was in rote learning as students in mathematics, for example, were required to memorize theorems and their proofs without the slightest deviation from the established text or methods.

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[1]Because of the change from the Julian calender to the Georgian to bring the fledgling Soviet Union into step with the rest of the world, the revolution is now celebrated on November 7. All the rest of the dates mentioned, even those prior to the revolution, are stated in accordance with this change.

In the middle of the 1890's, obligatory course structure at Moscow University was abolished [BG] and the selection of materials was left to the individual professors. Rather than the complete mastery of a university program, which had been the goal in the eyes of such people as the Russian physicist A. G. Stoletov, the goal became the attraction of students to scientific investigation. This was perhaps first evidenced in the laboratory of P. N. Lebedev who in ten years turned the university into a first class center of physical sciences. The change in mathematics was not as drastic and was first observed in 1901 in the lectures of Boleslav Kornelievich Moldzeyevski.

Boleslav Kornelievich[2] was born in Moscow on July 10, 1858, into the family of a professor of pathology and therapy at Moscow University. He completed his master's degree from Moscow University in October, 1886, and successfully defended his doctor's dissertation "On many dimensional manifolds" on January 20, 1890. Four months later he embarked on a one and one-half year scientific journey to Zurich, Paris--where he heard lectures by Darbou and Hermite--and Göttingen--where he heard Klein and Schwartz [R]. He returned to Moscow in February 1892 and began a teaching career which he continued until his death on January 18, 1926. Although basically a geometer, he was

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[2]In this paper, we shall refer to people by their full names, their lasts, last names with first initials, or as is the custom in the Soviet Union, their first name and patronym (father's name). For example, Pavel Sergeevich means "Pavel, son of Sergei." The patronym for a woman takes a different form. For example, Nina Karlovna Bari is the "daughter of Karl" (Karlovna).

interested in all areas of mathematics and was not a person to sit idly. Therefore during the summer of 1900 he began to prepare a course in the theory of functions of a real variable and the theory of sets. It was during this course which was first offered in 1901 that the terms "set," "power" (cardinality) of a set, "countable," and others were first introduced into the curriculum of Moscow University. This introduction of new material may be considered a first revolution in Russian mathematics and was not completely accepted by the students and colleagues of Moldzeyevski. The second and most renowned revolution was still fourteen years away.

The instigator of this second and most successful revolution was born December 9, 1883, in the town of Irkutsk in Siberia. He was the son of a merchant and the grandson (at least according to his father Nicholai Metrofanovich Luzin) of a serf of Count Strogonov [BG]. His mother was a sickly woman and from her Nicholai Nicholaevich Luzin probably inherited his poor health--a weakness which was to have a large impact on his life.

As a youth Nicholai Nicholaevich read widely in literature and in the sciences. He loved Jules Verne whose influence upon his imagination was immense. Pure philosophy gave his mind an abundant source of fuel. But mathematics was a problem: he was unable to memorize the required material and thus progressed poorly in both mathematics and languages. In the early 1890's his father moved the

family to Tomsk[3] to secure a better education for his only child.

Despite the efforts of Nicholai Nicholaevich, his grades fell because of his memorization problem and his health. His physical infirmities did not permit him to engage in the intensive study which was necessary to pass his examinations and, recognizing his problem, his teachers passed him on to higher courses solely on the basis of his classroom work. This inability to engage in intensive study to some degree affected his later work.

During his last year in the gymnazium, his poor standing in mathematics necessitated the acquisition of a tutor from the newly formed Tomsk Polytechnical Institute which had just opened that year. This student (whose name is forgotten) recognized Luzin's inability for rote memorization but, more importantly, recognized his innate ability for creative reasoning which proceeded from his lively imagination and was enhanced by his love of Jules Verne and philosophy. With proper coaching Nicholai Nicholaevich lost his hostility toward mathematics.

Desiring to be an engineer, in 1903 he matriculated in Moscow University in the Mathematical Department of the Physical-Mathematical Faculty where he wanted to strengthen

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[3]Later when Nicholai Nicholaevich matriculated at Moscow University, his mother and father accompanied him to Moscow. The first move was not an easy task as transportation from Irkutsk to Tomsk was only available by oxcart and river boat, although the move to Moscow was easier as the Transiberian Railroad's first line reached Tomsk in 1896. It is sad that this man who sacrificed so much for his son was eventually to lose all his savings on the Moscow Stock Exchange.

his weak mathematical background to enable him to pursue his goal. Initially he tried to enroll in the physics laboratory of N. A. Umov but was unable to because the class was filled. Thus, for the first time, he turned to mathematics and enrolled in the functional analysis course being given by Moldzeyevski and also became a central part of the student mathematical circle (i.e., study group) which at that time was under the chairmanship of the Russian mechanic N. E. Shukovsk. Luzin's ability at problem solving soon dominated the group and attracted the attention of Moldzeyevski and Dmitrii Fedorovich Egorov who was to become Luzin's mentor.

Born in Moscow, December 10, 1869, Dmitrii Fedorovich was the son of the Director of the Moscow Teachers' Institute [K]. He graduated from Moscow University in 1891 having completed his first scientific paper on the theory of numbers while he was an undergraduate. Kept on at the university as a postgraduate student, he obtained his master's degree in 1893, presenting his paper for the degree of Master of Pure Mathematics in 1899. He successfully defended his doctoral dissertation for the degree of Doctor of Pure Mathematics in 1901 (one of his opponents[4] was Moldzeyevski). Dmitrii Fedorovich spent the 1902-03 academic year abroad: the summer in Berlin, autumn and winter in Paris, and the following summer in Göttingen. He was elected to a professorship in the Department of Pure

[4]The term "opponent" refers to the practice in the Soviet Union (and in Russia at that time) of assigning two people to review the dissertation and to speak about it during defense.

Mathematics of Moscow University in 1903. Among other honors received during his lifetime were Corresponding Member of the Academy of Sciences of the USSR (1924) and an Honorary Member of the Academy of Sciences (1929). He died September 10, 1931.

In the spring of 1905 the political unrest festering in Russia burst forth in the University in the form of a strike, and studies stopped. Though they resumed in the fall, studies once more drew to a halt as auditoriums changed from lecture halls into places of mass agitation. The university became a beehive buzzing with discontent. Luzin himself was caught up in the movement and in order to continue his activities and not draw attention to himself and his family, he moved from the hotel quarters which he shared with his parents into the apartment of a widow and her daughter, Nadeshd Michailovna Maligen, who was to become his wife in 1907. The apartment on the second floor of 25 Arbat was to become one of the central points of the mathematical life of Moscow University. However, during the unrest it served as a safe house for fugitives and at times the space under Luzin's bed became a munitions depot [BG].

Distressed at the lack of opportunity for his student to continue studies at the university, and possibly also at the potential disaster which loomed before Nicholai Nicholaevich because of his participation in the revolutionary movement, Egorov convinced Luzin to continue his studies in Paris. Accompanied by V. V. Golubev (selected by Egorov primarily because of his speaking ability in French and German [BG]) Luzin left for Paris on December 1, 1905.

In Paris Luzin heard the lectures of Borel on the theory of entire functions and Picard on the analysis of series expansions of perturbed functions of celestial mechanics. The lectures of the latter had a profound effect upon Nicholai Nicholaevich because in them the living creativity of mathematics was demonstrated. In the College de France, Nicholai Nicholaevich heard Hadamard lecture on the theory of the diffusion of waves, and Darbou on the theory of surfaces. However his main work was studying mathematical literature at the Sorbonne, the National Library, and the library of St. Genevieve. It is said that he literally wore out the seat of his pants during his studies and often continued from the early evening until the sun rose [BG]. In his later life he said that creativity is not something which can be turned on and off but needs persistence. In this way he was much like Tschaikovsky who said that creativity does not just strike but is something one must work at. During this time in Paris, Nicholai Nicholaevich first began to form the basis for his later mathematical works.

Returning to Moscow during the summer of 1906, Luzin passed his graduation examinations and was kept on at the university by D. F. Egorov as a graduate student, but at that time he was not convinced of his mathematical abilities. Having become inspired to work among the people, Luzin began attending lectures on medicine during his free time. However he soon realized that the work in the anatomical theater was too difficult for him (again probably because

of the extreme amount of memorization required) and he began to attend lectures on philosophy, an abiding love from his youth. He ceased after a year when he realized that philosophy did not provide him a proper forum for truly creative work.

Returning to mathematics for the last time, in 1910 Nicholas Nicholaevich passed his master's examinations which brought him the right to teach in the university as a privat-dozent after the presentation of two trial lectures, one assigned and one of his own choosing. He presented these two lectures in the fall of 1910 and prepared to offer a course in the theory of functions. Upon learning that the right to teach this course had been reserved for S. S. Byushgens who had completed his examinations at the same time as Luzin, Nicholai Nicholaevich prepared to offer a course in integral equations. However again Egorov intervened, securing for him a two-year research fellowship in Göttingen and Paris in hopes that Luzin would complete work on his research paper, which was the remaining requirement for the master's degree.

At that time degrees in the Russian educational system were conferred by the universities. Later, under the Soviet government, all degrees were conferred through the government, specifically with the approval of the Higher Attestation Committee. There were then, as now, three degrees. The first degree, known as the university degree, required the successful completion of examinations and could or could not be accompanied by a paper. The second degree, the

master's degree, required examinations and a thesis of original scientific work and usually was obtained within one year after the university degree. The third degree, the doctor's degree, required an additional dissertation.

At the present time, there are still three degrees all of which now require papers. The second degree is known as the candidate's degree and usually takes three years and involves examinations as well as a dissertation; during the period of work on this degree the student has the title of aspirant, a term which came into being in the 1920's. The third degree which is still known as the doctor's degree requires only a dissertation and is currently obtained by only about ten percent of those people who earn the candidate's degree [KVL].

For more than a year Luzin worked in Göttingen, studying and listening to lectures. During that time, under the influence of Landau, he published his first paper in 1911 at the age of 28. This paper showed Nicholai Nicholaevich, who up to this time was not secure in his own abilities, that he did possess the creative abilities needed in mathematical research and spurred his work. In 1912 he proceeded to Paris and remained there until 1914 having received an extension to his fellowship, again at the insistence of Egorov who promised the faculty that Luzin would then be able to complete his work. In Paris he renewed old acquaintances (Picard and Borel) and made new ones (Hadamard and Lesbeque). He attended lectures and constantly continued his individual study.

Following his return to Moscow in 1914, the presentation of his thesis in May 1915 was so overwhelming that the Faculty of Mathematics requested that the University Council award him the title of Doctor of Pure Mathematics immediately.

In the archives of Moscow University there appears the following information about this request [BG]:

> May 13, 1915, in the Council of the University there was a presentation of the Physical-Mathematical Faculty dated May 13.
>
> On April 27 in a meeting of that faculty there occurred the public defense of the master's dissertation in pure mathematics of N. N. Luzin with the title "Integral and Trigonometric Series."
>
> The official opponents: Professor D. F. Egorov and Honored Professor L. K. Lachtin.
>
> The defense was deemed satisfactory and N. N. Luzin worthy of the doctorate in pure mathematics.
>
> The faculty petitions for the confirmation of N.N. Luzin in the degree of Doctor of Pure Mathematics.
>
> On the basis of Article 30, Section 1, page 3, of the University Statutes, it was resolved to give to N. N. Luzin, a candidate for the master's degree, the degree of Doctor of Pure Mathematics in view of the fact that his presentation of the dissertation distinguished itself to be of exceptional scientific merit and to give to him the necessary diploma.[5]

Thus Nicholai Nicholaevich Luzin attained for himself a secure position in the university and was free to concentrate on his lectures and his own research. In 1917 Luzin was elected a professor by the members of the faculty, a process which was abolished in 1918 by the first decree of the Council of the People's Commissars on university education which liquidated all academic degrees and titles. "Professor" and "reader" became merely appelations of appointments. However, university professorships were positions of exceptional distinction, even more so because there

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[5]This was something which had not been done in sixty years.

were so few of them. For example, in 1921 there were five professors of mathematics at Moscow State University, as it became known after the revolution; among them were Egorov, Moldzeyevski, and Luzin. A professorship was so influential a position that a piece of paper with a professor's signature was a powerful document in the hands of a student [L-II].

Freed from the pressures which securing his degree had placed upon him, Luzin turned his energies to his classroom presentations. His personal magnetism and the vivacity of his lectures, during which he demonstrated not only known results but involved his students in the creation of these results, drew about him an eager group of young students. One of them described his lectures in the following manner: "Other professors show mathematics as a beautiful completed structure--we can only admire it. Luzin shows it in its incomplete form, he awakens a desire to take part in its development" [L-II]. Luzin was an artful manipulator when seeking different proofs, directing his audience in securing a proof which he had developed, or sometimes saying that he had forgotten a proof and asking for assistance. Then efforts would be made at the blackboard by students who felt embarrassed when they failed but triumphant and envied when they succeeded.

In his classroom teaching and in his research, Nicholai Nicholaevich showed a remarkable intuition to see and guess new facts, and this ability played a strong role in the formation of his school of functional analysis. This ability

was something which Poincare valued particularly--"...the ability to find connections between apparently distant facts, between different branches of mathematics (here the ability to find physical meaning in abstract mathematical constructions is also relevant)" [BG].

The interest evoked by his lectures was not stopped by the end of the period and mathematical discussions continued in the hallways between periods, on the walk back to his lodgings on the Arbat, and even during evenings when his students gathered in his apartment. These followers of Luzin were a fun-loving and interested group who, in their search for an appropriate name with which to refer to themselves and their mentor, facetiously chose "Luzitania,"[6] a name known to everyone during the terrible years of the First World War. The names of many of these students are known to mathematicians in all areas, and the influence of their works then and later strongly affected the paths of mathematical investigation: M. Y. Suslin, D. E. Men'shov, A. Y. Khinchin, V. V. Veniaminov, N. K. Bari, L. V. Keldysh, A. N. Kolmogorov, M. A. Lavrent'ev, and L. A. Lysternik, who was to serve as the unofficial historian of the group. Working side by side with Luzin but not counted among the number of his "scientific children" were V. V. Stepanov and I. I. Privalov.

Included in the number also were Pavel Sergeevich Alexandroff and Pavel Samuilovich Urysohn, the founders of the Moscow school of topology, the first spinoff from "Luzitania" or the Moscow school of function theory. These

[6]The name first appeared in 1920 [L-III].

two men formed a part of what was called the "perfect kernel" of the "Order of Luzitania" which consisted of the first followers of Luzin: Alexandroff and Urysohn, V. V. Stepanov, V. V. Veniaminov, and four female students: N. K. Bari, Y. A. Rozhanskaya, B. I. Pevzner, and T. Y. Aikhenval'd. The later followers formed what was called the "imperfect kernel."

Because of the interest of Luzin in the continuum hypothesis, the mathematical achievements of the members of Luzitania were tallied by a hierarchy of "alephs." Initiates into the order received the title of aleph-nought and with each success--graduation from the university, first lecture, and so on--a "one" was added. Alexandroff and Urysohn received the high rank of aleph-five and Luzin himself the title of aleph-seventeen. This, and probably the entire hierarchy of alephs, evolved from one of Luzin's lectures[7] [La]. Arriving in class one day he announced: "The continuum hypothesis has been solved." Pausing for a moment of thought, Nicholai Nicholaevich wrote "aleph-seventeen" on the board. The classroom buzzed and then Luzin said "Why not?" It was all a joke.

The exuberance of the students for their teacher and their compatriots was expressed in the "Luzitanian March" [L-IV]:

> Our deity Lebesque,
> The integral our idol,
> Through rain and storm and snow
> We wend our merry way.
> . . . . .

[7]There are several versions of the origins of the "aleph" hierarchy, but this one seems the most plausible to the author.

We number seventeen,
All ready to devote our lives
To solving problems,
But there is nothing to solve,
Here Frechet solved,
There Lindelof proved...
Sad at heart sits
Aleph-seventeen.....

The foundations of the Moscow school of topology were laid during the summer of 1922 when Alexandroff and Urysohn collaborated in their renowned "Memoirs on compact topological spaces" which at present has versions in French (the original) and three editions in Russian. These two outstanding Soviet mathematicians were together constantly and were referred to among the members of Luzitania as the "two PS's," a reference containing a joke. In Russian there is a word "пёс" which means dog (male). The plural of this is "псы" while in Russian "PS's" is written "пс'ы." Thus within the confines of Luzitania, the two of them were referred to as the "two dogs." Their close friendship is also attested to by the fact that Urysohn affectionately called Alexandroff "пусы" which is the Russian way of referring to someone by his first two initials, in this case "P.S."

Pavel Sergeevich Alexandroff,[8] the older of the two, was the first to arrive at Moscow University. He was born May 7, 1896, in the town of Bogorodsk, the son of a country doctor who at the time was the senior doctor at the Bogorodsk District Municipal Hospital. In 1897 the family moved to Smolensk where Sergei Alexandrovich Alexandroff became the

[8]Pavel Sergeevich Alexandroff died November 16, 1982, after a prolonged illness.

senior doctor at the Smolensk Provincial Municipal Hospital until his death from typhus in 1920. The Alexandroff family consisted of the father, mother, two daughters, and four sons, of whom Pavel Sergeevich was the youngest child.

From his youth, Pavel Sergeevich was very interested in the sciences, first in geography and then geology, and later an even greater interest in astronomy. In school he came under the influence of Aleksander Romanovich Eiges, his mathematics teacher, and despite a careless inattention to signs reflected in his papers and the difficulties he had with factoring during his early study of algebra, Alexandroff was inspired by his teacher. The fours he received on his algebra papers changed to top scores of five when he began to study geometry and the non-euclidean geometry of Lobachevskii became as "enchanted kingdom" for him [A-III]. His relationship with Eiges also became social and in later years his brief marriage to Ekaterina Romanovna Eiges, sister of Aleksander Romanovich, ended when he became convinced that he was unfit for a domestic life.

Pavel Sergeevich enrolled in the Mathematics Department of the Physical-Mathematical Faculty of Moscow University in the fall of 1913 and found the first year's mathematics course somewhat staid and ill-presented. However, he soon found solace in the reading room of the university library where he read Cantor's memoirs on set theory and discovered in the world of transfinite numbers a topic as entrancing as non-euclidean geometry.

Alexandroff's friendship with Luzin began in 1914 upon Luzin's return from his second trip to Paris and Pavel

Sergeevich was fantastically inspired by the extraordinarily powerful impression he made. Alexandroff stopped him after class for advice concerning his plan of study and was struck by the careful attention which this renowned young scholar paid to an eighteen-year-old student. Having heard him out, Nicholai Nicholaevich skillfully ascertained Alexandroff's desires and then outlined the general directions which he felt would be most appropriate for Alexandroff's studies. Then subtly--without pressure and, retrospectively in Alexandroff's opinion, correctly--he coerced the young Pavel Sergeevich to choose a particular one of these directions. Thus Alexandroff became a pupil of Luzin during a time which he regarded as the era of Nicholai Nicholaevich Luzin's highest creative development [A-I].

In the spring of 1915, Luzin presented Alexandroff with the task of solving the problem of the cardinality of the class of Borel sets. This Pavel Sergeevich accomplished during the summer of 1915 by a method which became known somewhat controversially as the "A-operation." When Alexandroff first revealed his operation and the plan for its subsequent use to him, Luzin did not think that it would work (and neither did Cartheodory when Alexandroff and Urysohn explained it to him in 1923). However, fortunately Alexandroff disregarded his teacher's advice and successfully solved the problem using his own method.

He presented his results in a lecture to the student mathematical circle on October 13, 1915 (a lecture attended by Sirpinski who had just arrived in Moscow). It was at this time that Alexandroff first met Suslin and in a

conversation following this talk, it was proposed by Suslin that the operation be called the "A-operation" and the sets obtained by its application to closed sets "A-sets" in honor of the discoverer in just such a manner as the terminology of "B-sets" after Borel.

The following summer Suslin successfully undertook the problem of showing that every A-set is not a B-set, a result which was reported in the only mathematical paper he ever wrote. Alexandroff had already shown that every B-set was an A-set but had been unsuccessful in determining the converse. During the summer of 1924 while visiting Hausdorff in Hamburg, Alexandroff proposed that the sets he discovered (the A-sets) be called "Suslin sets" since Suslin was the first mathematician to prove that these sets were really new. This proposal, hesitantly supported by Urysohn who probably felt that the honor of the name should belong to its discoverer, was accepted and used in the next edition of Hausdorff's "Mengenlehre." One reason for this change was that Luzin had taken to calling A-sets "analytic sets," despite the fact that he knew the reason for the name. Alexandroff points out that by that time his friendship with Luzin, which had once been close and sincere, was estranged [A-III].

It was also during the meeting of the student mathematical circle on October 13, 1915, that Pavel Sergeevich first met Pavel Samuilovich Urysohn. The latter was born in Odessa on the Black Sea January 22, 1898, the last of four children and only son of Samuil Josephovich. His three sisters

(Anna, Lena, and Lina) were considerably older than he and as a youth he had few playmates his age and thus began studying earlier than most children. Chemistry and physics were his particular favorites and his first scientific publication was in physics.

His mother died in 1909, and at the insistence of Lena, Pavel Samuilovich, his father and sister Lina moved to Moscow in March 1910 where they were joined shortly thereafter by the oldest sister Anna who had been living in Poltava. Pavel Samuilovich yearned for adventure which is illustrated by the fact that he toured Moscow on foot within a few days of his arrival.

After the death of his mother, the youngest sister Lina became a mother to him and in later years she was to write a book about him for school children [N]. She travelled abroad with him and this further instilled in him a great love for travel and adventure which was later to take him and his close friend Pavel Sergeevich on two scientific journeys. In the summer of 1911, he and Lina visited Finland, and during the summer of 1912, Germany, Switzerland, and Pre-War Austro-Hungary. Pavel Samuilovich was a good linguist, having studied French as a youngster in Odessa and continuing to study French and adding German in the gymnazium he attended in Moscow.

Urysohn enrolled in the Physical-Mathematical Faculty of Moscow University intending to study physics but fell under the aura of Luzin and switched to mathematics.

Like Luzin, Pavel Samuilovich drew a crowd of people around him and for many of the same reasons--he was the

senior of the Luzitanians not in age but in scientific results. Lazar Aronovich Lysternik described him in the following manner [L-III]:

> In him was everything which "attracted the heart": pleasant appearance, open character, benevolent relationships with his associates, multiplicity of interests and great talent. But the main secret of his charm lay in the combination of scientific maturity and youthful spontaneity, and in his attractive smile there was something child-like. Pavel Samuilovich was a cheerful man, full of life.

While his character revealed this charm to others, his personal thoughts expressed something much deeper--an unhappy childhood and a sense of foreboding. On January 4, 1918, Pavel Samuilovich started a diary which he kept for several years until he abruptly stopped it on March 19, 1921, and in it are revealed his personal thoughts about his life as well as his work [LL].

Within the diary are two excerpts which reveal the inner Pavel Samuilovich, only the date of the second of these entries is known [LL].

> I had no childhood, since I began to be busy at an age when I had hardly finished infancy. I had no adolescence since my contemporaries were away and I did not know any games. I had no youth since I did not know the most important thing--love. Finally there will be no old age since I shall die young.

> 4234 [April 23, 1919, Wednesday]. I decided a long time ago that the entry in the encyclopedia will begin thus: Urysohn, Pavel Samuilovich (1898-1935, mathematician, etc.

> The external appearance of the "diary" could not be more modest. It consists of the usual student exercise books with squared paper. There are four of them, a fifth scarcely begun. They are written in a very small handwriting with a very sharp pencil and in such microscopic letters that they look as if they were made by a sewing machine. There are also certain peculiarities of the author's own invention: a) he wrote without vowels; b) he invented a shorthand for

> dates; c) his journeys (whether on foot or by other form of transport) are recorded, as was common at the time, in versts.[9,10]

The shorthand he used for dates was as follows [LL]:

> The first one or two numbers denote the month, the next two the date, and the last number the day of the week; note that 0 corresponds to Sunday, 1 to Monday and so on. For instance, 10261 means October, 26, Monday.

Within Luzitania, Pavel Samuilovich found his lost childhood and the childhood friends that he did not have. During a visit to Petrograd (now Leningrad) to celebrate the centennial of the birth of Chebyshev in June 1921, the young members of Luzitania wandered through the gardens of Peterhof (a home of Peter the Great, now called Petrovorets) north of Petrograd enjoying the wonderful sight of the many fountains which were set working for their benefit. Suddenly a heavy rain began to fall and puddles quickly formed [L-IV].

> Yulya Rożhanskaya bravely took off her shoes, the majority of young men and women followed her example. I remember with what an expression of childlike delight Pavel Urysohn splashed through the puddles. He was wearing a long overcoat, and when he rolled his trousers to the knees it looked as if this overcoat was all he had on, and the phrase "Urysohn's Peterhof Coat" made its appearance in Luzitania.

While Urysohn was engrossed in his undergraduate studies and research, Alexandroff was having a difficult time coping

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[9] For many years the diary was in the possession of his sister Lina and parts of it appear in the book she wrote about him [N], and elsewhere [L-III], and [LL]. The diary no longer exists, destroyed by his sister before her death. The author has been told that she did so because she feared that parts of it might reflect badly upon his memory.

[10] 3,500 English feet, or 1.06 kilometers.

During these times, the first tragedy befell Luzitania. One of the students who had accompanied Luzin was Suslin but he did not fare well in Ivanovo and lost his position. V. V. Golubev and I. I. Privalov tried to get Suslin a position at the University of Saratov, for which a letter of recommendation was expected from Luzin. He refused to write it. Suslin returned to his home (in the Province of Saratov) where he caught typhus. In a letter to his friends in Ovanovo, he expressed his opinion that he would not survive and he did not, dying in 1919. It is reported that Luzin felt responsible for the death of this promising young man, and for the rest of his life he kept a portrait of Suslin on his desk. Although Suslin published only the one paper on A-sets, he left his mark in mathematics, for this topic spawned much mathematical literature and is even now an active topic of investigation.

However, the time spent working in Ivanovo (1918-1922) did not daunt the growth of Luzitania because Luzin made frequent trips to Moscow on the institute's railroad coach and, in fact, transferred the majority of his work to Moscow in 1920. Every time the news of Luzin's arrival spread like wildfire throughout Moscow State University, the life of Luzitania again began to seeth and work commenced again in the seminar at the university and, during the evening, in the hospitable confines of Luzin's apartment on the Arbat.

The difficulties encountered during this time and the trials of acquiring food are clearly evidenced in the diary of Pavel Samuilovich Urysohn in his description of a six-day

with a failure in his mathematical prowess which, in retrospect, is quite understandable. After his initial success with the Borel set question, in 1917 Luzin gave him the general formulation of the continuum hypothesis. Although he did obtain some good results as it turned out (one of which was later included as a supplement to the famous Memoirs), Alexandroff was discouraged [A-III].

> It became clear to me, however, that my work on the continuum problem had ended in a serious catastrophe. I also felt that I could not go on, so to speak, to further things in mathematics, and that I had to come to a decision about my life.

Motivated by a letter from Sasha Bogdanov, a schoolmate, Alexandroff decided to leave Moscow and go to Novgorad-Seversk where he and Sasha became involved in setting up the Chernigov Soviet Dramatic Theatre, Sasha as an actor and he as a producer, as well as giving lectures on the theatre and literature.

The first years after the revolution were tumultuous ones in all of the new Soviet Union, especially following the difficult times of the First World War. Fuel and foodstuffs were scarce in Moscow and, as a consequence, Luzin and many of the other professors of Moscow State University sought positions elsewhere. Following the October Revolution, the Soviet government quickly established an ever-expanding network of higher educational institutions, enabling Luzin and several others--including a number of his students--to secure positions in the Ivanovo-Vosnesensk Polytechnical Institute which was established in Ivanovo-Vosnesensk (now Ivanovo) in 1918.

journey to a region outside Moscow to get food. Pavel Samuilovich considered such journeys necessary not only for the procurement of food but also for the benefits of the trip itself. The following description of this trip is paraphrased from his lengthy account in the diary (the journey began on Friday, October 4, 1918) [N].

It was difficult not only to obtain tickets on a train but also to obtain permission to make such a journey. Vasya Popyslkovskii received permission to undertake such an excursion for four people and invited Pavel Samuilovich to go along. Because the possibility of acquiring tickets on a long-distance train was remote, they decided to purchase tickets on a local. Even this necessitated waiting until midnight on Friday, sitting on the filthy floor of the station. Having finally purchased tickets, the four hurried to the platform and encountered another two-hour wait on a cold and muddy platform. There were about one thousand people waiting (men, women, young and old) all carrying baskets to bring back the food which they hoped to obtain. Finally a train consisting of fifty-three cars arrived.

The people stood up and, as one, tried to get on the train and a wild fight ensued. The press of bodies upon the narrow entrances to the cars made the process frightfully slow. The air was filled with oaths and cries with women causing most of the difficulty: they made their way stealthily by crawling through legs or violently by pushing between men, constantly screaming. Urysohn felt that the noise must have been heard far from the confines of the Kurskii train station on the east side of Moscow.

Finally Pavel Samuilovich acquired a place in a car becoming separated from his basket in the process and later discovering that someone was using it as a seat cushion and that it was hopelessly broken. He found a place by a wall in a car containing some one hundred fifty people where it was possible to sit only by drawing legs close up to one's chest. For a while this position was fine but then legs grew numb, and it was necessary to stand and run in place to restore circulation. For part of the trip Pavel Samuilovich sat in the doorway, dangling his legs outside, which was bad because of the cold and contributed to his problems later in the excursion. As the day progressed, the crowd thinned out at various stations and toward evening he was able to stretch out on the floor and sleep, using an empty suitcase as a pillow.

Urysohn calculated that the train covered only about thirty versts[10] that day but the slowness enabled him to enjoy the beautiful scenery through which they travelled. On Sunday the four companions detrained and travelled to a small village a distance of some 6 versts from the town of Polol'sk where they had alighted. In the village they purchased 12 poods (432 pounds) of food which they loaded on a rented farm cart pulled by a horse.

The journey to the village and the resulting purchases had taken all day and it was evening when they finally started back for the train station. It began to rain and

[10] 3,500 English feet, or 1.06 kilometers.

when the road turned to mud, they had to help the horse by pushing the wagon up a hill. It was at this juncture, perhaps, that Pavel Samuilovich lost a sweater which he had thrust into his jacket.

Reaching the station they unloaded their goods and collapsed. It had been decided that when the train arrived they would load their goods on the roof of a car to avoid the crowded conditions inside. However, when the mail train arrived (which was at three in the morning instead of the scheduled eleven at night) they forgot their plan and ran back and forth trying to find a place, having been assured that there would be space. They found the car vestibules crowded with people and locked. By the time they remembered their strategy, the train left without them. Therefore they went to sleep on the platform using sacks of flour as pillows. Everyone except Pavel Samuilovich slept well--he was worried about his sweater.

Consequently he got up at five and went to look for the sweater, but to no avail. He hiked all the way to the village and back without discovering it. The rain and the cold took their effects and he became ill. When the next train arrived Monday at two in the afternoon--two hours late--the four travellers loaded their belongings onto the roof of a car, Urysohn unable to help because of his illness. Having secured their belongings, they settled themselves on top of the car wrapped in a friend's rug. To his joy, he had a three-sided panoramic view of the countryside and the return trip to Moscow would have been extremely pleasant for him except for the civilian authorities.

In Skuratov, the next town, they heard shots from rifles and were told by two militia men--one on the ground and one walking the roofs of the cars--to get off the roof of the car. Obediently they pulled their belongings down. Arguing with the fellow on the ground (the other having proceeded with his patrol), they convinced him that they were Soviet employees and had to return as scheduled in order not to have problems with their jobs. They then commenced to reload their possessions. When they were almost finished with this task, the rooftop guard spotted them and said one word: "Down!"

He was adamant and the other vaccillant (to permit it was not proper but it was also not right to force them to lose their spot on the train) but strength prevailed and the goods were once again unloaded.

Physically exhausted they sat in the station waiting for the next train which was due at eight in the evening. Finally at ten a train appeared. It pulled into the station and everyone stood up with their possessions and the train continued past. The train returned, left, went back and forth several times, and finally stopped nearly a verst from the station. Little by little the people carried their belongings to the train. Then at midnight just as they were all aboard, the train pulled up to the platform and left shortly thereafter.

As the train progressed toward Moscow, it gathered more passengers and space became scarce, but recognizing that he was sick, people provided sufficient space for

Pavel Samuilovich to lie down. The trip from that point took thirty hours during which he was in a semiconscious state because of a high fever. Arriving in Moscow early Wednesday morning, they discovered that the railroad had arranged another "swinish trick."

To pay for the privilege of taking their goods aboard, the passengers were made to get off on a bridge and stand in line to pay a tariff. There were several thousand people and the process was slow, the weather cold, and the baggage froze to the ground necessitating chopping it free in order to move it slowly forward. After one and one-half hours, Vasya took pity on Pavel Samuilovich and sent him home on a trolley, bringing his share of the purchases by cab later. Urysohn spent three days recovering from his illness, too weak to work.

These times also had ill-effects on Alexandroff. Although the revolution had succeeded in toppling the tsar and his government, military forces supporting the tsar still fought on. In October 1919, Deniken troops moved north and occupied Chernigov. Alexandroff's friend Sasha had been an officer during the war and he was immediately recalled to active duty. Shortly after that Pavel Sergeevich was arrested. For a few short hours his friend Sasha managed to be appointed as his guard in a detention center and then he was taken to prison. The military handled his case and several times during his week's imprisonment he was interrogated.

When Alexandroff asked of what he was accused, his interrogator told him that he had voluntarily, actively, and energetically collaborated with the Bolsheviks, and thus actively supported the revolution and contributed to the popularity of the new Soviet government. Supposedly Pavel Sergeevich had given public lectures to this fact in addition to lectures in agitational education. One interrogation period ended with him being told, "If the Russian intelligentsia behave like you, the Bolsheviks will still govern Russia six years from now" [A-III].

Some influential people who had heard his lectures intervened on his behalf, his case was transferred to civilian authorities, and he was released to await trial. However, shortly thereafter the Soviet army reoccupied Chernigov and the onus of a trial was removed.

In November 1919 Pavel Sergeevich presented some lectures on mathematics in the newly opened Institute of Mathematics in Chernigov. After recovering from a six-week bout with typhus, he made up his mind to return to mathematics and travelled to Moscow where he felt he was welcomed as the returned "prodigal son."

It was decided that he should prepare for his master's examinations at once and he drew up an outline for his work. It happened that Urysohn was also taking his examinations and had essentially the same tests so they started studying together, attending symphony concerts, and becoming fast friends during that winter of 1920-21.

The following is the syllabus of L. A. Lysternik for his master's examinations in 1922 and is probably similar to that of Alexandroff and Urysohn. There were sixteen examinations and the recommended references are listed in parentheses [L-II].

1. Classical analysis (based on the third volume of Picard's well-known analysis course).
2. Theory of functions of a real variable (books by Borel and Lebesque).
3. Theory of functions of a complex variable (Osgood's book).
4. Ordinary differential equations (Goursat's second volume...)
5. Partial differential equations (Goursat...)
6. Integral equations (Goursat's third volume...)
7. Calculus of variations (the course by Bolza and Tonelli).
8. Equations with functional derivatives.
9. Geometry 1 (Hilbert's Foundations of Geometry).
10. Geometry 2 (Bianchi's massive course of differential geometry...)
11. Theory of numbers (Minkowski's Diophantine Approximation).
12. Algebra (Weber's course).
13. Theory of invariants.
14. Theory of probability.
15. Mechanics 1 (general, Appell's course, volume 1).
16. Mechanics 2 (continuous medium; Appell's other volume and also Zhukovskii's article "Modified Kirchoff methods" on streamline flow).

It should be noted that many of these courses are presently part of the undergraduate curriculum in the Soviet Union and that at that time most of the texts used were either in French or German requiring a good knowledge of those languages on the part of the student. Also, up to the revolution, French was spoken by many of the higher social class status Russians, particularly in Petrograd. Also an examination on any subject could be replaced by a thesis on that topic containing a new result, a new proof, or something similar.

The rapid growth of knowledge, in mathematics and elsewhere, during the early years of this century found itself reflected in a change of the length of time a student studied. The four-year university course became a four-and-one-half year and later a five-year course. The nine-year secondary schools of that time became ten-year and, even for a short period, eleven-year. Students in the Soviet Union start school a year later than in the United States but they go six days a week from September 1 through May 30 which is approximately the same secondary school time as the current twelve years in American schools.

During the summer of 1921, a group of Luzitanians rented a dacha for the summer in the village of Burkovo on the banks of the Klyas'ma. It was here that one morning while bathing in the river, Urysohn revealed to Alexandroff his results on dimensionality, a mathematical discussion continued in the same place until the late afternoon.

It had been during June of that same summer following the successful completion of his master's examination that Professor Egorov had posed the problem for him: to determine an intrinsic definition of dimensionality which would be consistent with the accepted concepts in the line and place and which would be applicable to surfaces. Pavel Samuilovich spent the entire summer working on this problem, considering a version and then discarding it as he would construct an example to show that it was unsatisfactory. Thus passed the entire summer until that morning in August when he related his finished definition and consequential

theorems to Alexandroff. The paper which Urysohn outlined during that memorable day was written during the winter of 1921-22 and completed in the spring of 1922. Entitled "Memoirie sur les multiplicities cantoriennes," it was submitted to Fundamenta Mathematicae where it was accepted and published, because of its length, in volumes 7 (1925) and 8 (1926) after Urysohn's tragic death.

The following year, the two PS's rented a room in another dacha near the first one and began to work together on topology, the result being the famous "Memoirie sur les espaces compacts." In the preface to the third Russian edition of this work, Alexandroff describes the background of previous work and the thoughts which motivated the two of them [AU].

> The founders of the theory of topological spaces (general topology) were, as is well known, Maurice Frechet and Felix Hausdorff (1868-1943), despite the fact that neither one of them, in my opinion, recognized in his own works the creation of a new mathematical discipline.
>
> Frechet introduced to mathematics (in his fundamental work...in 1906) the fundamental concepts of metric space, its compactness and completeness, for as yet the concept of topological space did not exist.
>
> Motivated in his investigation by the great analysts of his time, the first and foremost being J. Hadamard, Frechet (as is seen even from the title of his work--"Sur quelques points du calcul fonctionnel"--) regarded his investigation as a component in the just-arisen functional analysis and not as the construction of a new set theoretic discipline. Even in the ensuing monograph of Frechet--"Les espaces abstraits" (1925)--his attitude toward things had not changed: This was a catalogue of large numbers of "abstract spaces" rather than a systematical theory of something especially important which was common to them.
>
> If Frechet proceeded out of necessity from mathematical analysis in the broadest sense of the word, then Hausdorff stood strictly on the basis of the theory of sets. First of all his interest was in the logical structure of point sets as it existed at the end of the

> first decade of the current century. By means of his own important four axioms of a topological space, he hit the target, while giving together with a large collection of different possible constructions a sufficiently broad, and at the same time really "vigorous," class of spaces, now well known by the name Hausdorff. The four axioms and the subsequential addition to them of the axioms of countability permitted Hausdorff in his splendid book "Grundzuge der Mengenlehre" (1914) sufficient latitude to solve the problems of logical analysis and the axiomatical propositions of the theory of point sets.
>
> In this Memoir, P. S. Urysohn and I stood entirely on the basis of the Hausdorff "theory of sets," but the distinction from Hausdorff was that we were interested not in questions of the logical analysis of classical theory, but within the newly discovered topological spaces, we saw fascinating new topics of mathematical investigation, and we decided actually to undertake this investigation with all possible systematicness, beginning with that end which seemed to us the most promising--the concept of compactness.

Their work continued into the autumn of 1922, climaxing with their proof of the general metrization theorem although they felt that it deserved immediate publication and for that fact did not include it with the Memoirs.

Upon completion of their work, they began preparing the manuscript and completed it in January 1923. The paper was submitted and accepted for publication in Fundamenta Mathematicae but was not published until 1929 and in another journal (see below page 364).

During this period of preparation, Urysohn decided that they should go abroad to Göttingen the following summer. Needing money to finance this trip, they decided to offer public lectures on the theory of relativity, a cycle of four lectures repeated in different auditoriums in Moscow.[11] The first cycle of lectures was given on January 12, 14,

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[11] These lectures were repeated at least once prior to their 1924 trip in the town of Voronezh (about 500 km south of Moscow) in March 1924.

16, and 17 and were announced by means of the following poster [N]:

> On the mathematical knowledge of the world in light of the Theory of Relativity. The lectures do not assume any special mathematical training on the part of the audience. All lectures begin at 8:00 in the evening. Tickets are being sold at the ticket booths of the Petrovski and Tverski theatres and on the day of the lecture at the entrance from 5:00 in the evening.

The lectures were the financial success they had hoped and they obtained the needed money.[12] Besides acquiring the money, there was much other work to do: They corresponded with mathematicians in Göttingen whom they knew only by mail, in order to arrange to be met upon their arrival, and to have lectures scheduled. They also had to obtain important documents. The following excerpt is from a mandate numbered 3587/2457 and dated April 4, 1923 [N].

> The National Commissionariat for Education herein certifies that the bearer of this, a senior research officer of the scientific research institute of mathematics and mechanics, Pavel Samuilovich Urysohn is dispatched to Germany for a period of four months for scientific study.

This permission was followed by the usual words soliciting for Pavel Samuilovich all kinds of assistance in the successful completion of this mission. The other important document from the department of visas of the Deputy People's Commissar for Education, dated April 28, number 1024, urged the entry into Germany of research fellows P. S. Urysohn and P. S. Alexandroff.

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[12] At that time, the Soviet currency was exchangeable internationally. However, at the present time the value of the rouble is only within the Soviet Union where its exchange rate is artificially controlled by the government.

Similar difficulties obtained for ordinary travel even within the Soviet Union. In June 1921, the 100th anniversary of the birth of the great mathematician P. L. Chebyshev was to be celebrated in Petrograd. An invitation to participate was sent Moscow University from which he had graduated and to the Moscow Mathematical Society of which he was one of the founders. Active communication between the mathematicians of Petrograd and Moscow had broken off at the beginning of this century and thus the invitation was of a rhetorical nature, but in Luzitania a spontaneous decision was made to attend the conference with the most troublesome aspect being transportation. They decided that such group travel could be accomplished most satisfactorily with a private railroad coach. Vladimir Nicholaevich Veniaminov--who worked in the Transport Institute and had the necessary contacts--was put in charge of this project and he enlisted the support of Otto Yul'evich Schmidt. With proper influence, they accomplished the task [L-IV].

The final requirement was to obtain permission for the people themselves, which they accomplished by writing (on a long sheet of paper) a single certificate for all the travellers, to which was applied the seals of the university and all the other organizations for which they worked. It should be noted that at this time, Luzin held the title of professor, a title which bore much influence.

Although the trip created much excitement among the travellers, the feelings of the two PS's as they prepared for their first scientific trip were greater still but mixed

with their joy were feelings of apprehension. They were to be the first Soviet mathematicians to travel outside the country since its founding and they felt that theirs was an extremely important mission: to prove themselves as Soviet topologists. They were determined not to disgrace themselves either in their knowledge of foreign languages (Alexandroff was extremely fluent in German having studied it from his youth) or in their manners. In the latter respect, they even had calling cards printed in scholarly German according to proper etiquette.

The summer at Göttingen was a complete success, their lectures were well received and they made friendships with Hilbert, Courant, Landau and Noether which were to last a lifetime. During this summer, Urysohn and Alexandroff presented lectures to meetings of the Göttingen Mathematical Society and wrote four papers[13] which were submitted to Mathematische Annalen. Despite the recommendation of Hilbert, who was the official editor, Alexander Ostrowski, a Russian who had left the Soviet Union after the revolution--at that time a privat-dozent at Göttingen--and the man to whom the papers were properly presented, did not like them and put them aside. It was not until they returned to Göttingen during the summer of 1924 on the second of their journeys that action was taken. Then supported by Emmy Noether, L. E. J. Brouwer, the member of the editorial board in charge of topology, and Hilbert, the papers were published

[13]Three of these were announcements of results which were to appear in the "Memoirs."

at the end of 1924, but only after Urysohn's untimely demise.

Similar also is the story of the publication of the Memoirs which is related by Alexandroff in the preface to the third Russian edition [AU].

> The delay with the publication of the perfectly prepared text of this Memoirs occurred in the following manner. Since the journal Fundamenta Mathematicae shortly before this had received the extensive work "Memoire sur les multiplicities cantorienes" of P. S. Urysohn, the printing of which was stretched out in two volumes (the seventh and eighth) naturally the editor of Fundamenta was not able to proceed immediately with the printing as well of the extensive "Memoirs on compact topological spaces." As a consequence of this, at the initiation of L. E. J. Brouwer, the question of transferring publication to another journal was raised. However, everywhere such a big work caused difficulties. That's why, indeed after the death of Urysohn, because of the stringent insistence of Brouwer, the work was published in Verhandelingen der Kon. Akademie van Wetenschappen te Amsterdam where publication (because of the necessity of sending galley proofs from Amsterdam to Moscow and back again) was further delayed another three years (1926-1929). As a result it occurred considerably later by comparison had it been kept in Fundamenta.

The Memoirs is dedicated to D. F. Egorov. The reason for this is interesting but also shows a failing of the mathematical comprehensiveness of Nicholai Nicholaevich Luzin [A-III].

> In our work on general topology, we had a great deal of support from Egorov. Luzin had once said that he considered the theory of topological spaces to be part of mathematics of little interest and quite useless, like ideal theory. Egorov objected strongly and even a little sharply, saying that he had always regarded ideal theory as important and necessary, and that he felt the same way about the theory of topological spaces. With that the conversation ended. Urysohn and I dedicated our "Memoirs sur les espace topologiques compacts" to Egorov, and this dedication is printed on the first page of the original French edition of this Memoirs and was reproduced in the preface to the Russian edition which was published as a separate monograph.

During August of 1923, the two PS's went on a walking tour of Norway, again at the instigation of Urysohn. Both young men were similar in many respects: both loved music and both loved bathing (and this is different, in Alexandroff's opinion, than swimming). Both had learned to swim at a young age, Urysohn in the Black Sea and Alexandroff in the River Dniepr.

Music played a large part in both of their lives and Urysohn expressed his feelings about music in the following manner [N]:

> Music and mathematics are united. Music itself is emotional from the arts. It expresses feelings and appeals to the senses. Mathematics is the most intellectual among the sciences, it appeals to the mind, to the logical principles in a person. These contrasts come together and interact on each other. Music assists a mathematician in his creative work. The most abstract of arts helps the most abstract of the sciences. In the harmony of mathematical proofs there is poetry and music.
>
> People perceive music in different ways. Some enjoy the beauty of sounds, the harmonies of melodies; they perceive feelings and thoughts enclosed in these melodies, rhythms, forms. To others music calls forth more concrete notions and memories.

Alexandroff's brothers and sisters played musical instruments, violins and the piano, and as a youngster he would study while they practiced. During the winter of 1920-21 while the two PS's studied together for their master's examinations, evening concerts became a daily routine in their lives. In fact it was after an evening concert when they dallied on their way home taking turns walking each other to his door that Alexandroff felt was the moment that sealed their friendship.

It would appear that Urysohn was the more athletic of the two and undoubtedly it was he who, during their visit with Hausdorff in Bonn in the summer of 1924, suggested the daily, exceedingly difficult, swim across the Rhine, as well as a late night winter plunge in the Moscow River to win a bet.

Alexandroff's athletic inclination was undoubtedly hampered by his poor eyesight, a handicap from his youth, which deteriorated with each passing year. In fact he has been blind since late 1979. A humorous anecdote about his blindness reflects the fact in his later years, his vision range was at most four meters, Alexandroff did not wear swimming trunks when bathing, preferring to go "au naturel." As he would explain to his students on some of their outings around Moscow in later years, "If you don't see any women in the area, then there is no need for clothing."

His poor eyesight was a blessing in other ways. He suffered from claustrophobia and his weekly topology seminar is held on the fourteenth floor of Moscow State University which requires the use of a usually crowded elvator. This didn't bother him because he could not see that he is enclosed in a small area.

It was undoubtedly Urysohn's love of the athletic life and his love of swimming which led to his unfortunate death in August 1924. After a visit with Brouwer in Holland following a second sojourn of several months in Göttingen and some time with Hausdorff in Hamburg, the two proceeded to Paris and thence to the small town of Batz on the southern

coast of Normandy. They sought a quiet and peaceful place in order to continue their mathematical work. It was there on a peaceful Sunday with a somewhat stormy sea that they went bathing in the late afternoon.

In Batz there is a cove which is sheltered from the ocean by two rocky craigs. They started bathing there and then Urysohn led the way out of the cove and into the ocean. Suddenly, seemingly out of nowhere, a huge wave arose sweeping Alexandroff into the cove and up on the beach. Momentarily stunned, upon regaining his senses he saw Urysohn, apparently unconscious, still in the water in a sitting position, rocking with the waves. Swimming out to him Alexandroff grasped his body and began to tow him to shore.

Although there were people on the shore, no one came to his assistance until he was quite close, whereupon he was thrown a rope and pulled to the beach. People gathered around and some of them attempted to revive Urysohn by means of artificial respiration but it was in vain as the waves had flung him against the rocks which guarded the entrance to the cove. One in the party was a doctor and when Alexandroff, believing that Urysohn was still alive, asked him what his course of action would be, the doctor replied, "Que voulez-vous que je fasse avec un cadavre?" or "What do you want me to do with the corpse?" Alexandroff was stunned and could only contemplate upon the grammatical construction of "fasse" derived from the verb "faire" which the doctor had used.

When he recovered his composure somewhat, he telegraphed his brother Sergei in Moscow: "Prevenir doucement Lina, sans dira pere: Paul enoye en se-baignant." or "Carefully inform Lina, without telling her father: Paul drowned while bathing" [N]. Both the Urysohn family and the entire mathematical community of Moscow were stunned. This was about all that they knew about the tragic accident until Alexandroff returned to Moscow in early September. Trying to follow what he felt Urysohn's father would have wanted, Alexandroff found a rabbi and arranged for a Jewish burial in the village's small cemetery.

Thus the love for adventure and the unknown of the world led to the death of a fine young mathematician and ended what had been a brief, but illustrious, mathematical career. It is difficult to predict to what heights Urysohn might have risen for he was only beginning, but he had shown uncommon insight and intuition in many of the mathematical problems he investigated and these problems covered a wide scope of interests.

The deep affection for and the respect of the mathematics of Urysohn by his comrades is clearly evidenced by the efforts of his friend V. V. Stepanov [L-III].

> His close friend Urysohn told him of his own work (on the properties of solutions of a class of functional equations). After Urysohn's death no records of this work remained. Stepanov completely restored this work, which contained a number of subtle points, gave a lecture on it to the Society, and prepared it for press. Thus this paper was rescued and went into the collection of Urysohn's works.

Alexandroff luckily escaped harm in Batz, but his love for bathing led him into harm's way at least once again.

Attending a small conference in Tiraspol in 1972, he was enjoying a swim in a river when a power boat approached him. Realizing that the driver did not see him, Alexandroff took the only avenue of escape which existed--the depths of the river. However, the boat was faster than he and he was hit by it and hospitalized with a broken pelvis.

Following the death of Urysohn in Batz, Alexandroff returned to Göttingen where he was met by several Soviet mathematicians and L. E. J. Brouwer. Having in his possession works and photographs of Urysohn made by Brouwer during their visit with him, Alexandroff sought to insure their safe return to Moscow by requesting and receiving the following document [N].

Ministry of Education
Berlin, September 2, 1924
No. 327/325

CERTIFICATE

This certifies that Professor P. S. ALEXANDROFF is returning to his place of his permanent work in Moscow and is bearing with him manuscripts in mathematics, separate off-prints of articles, photographs and scientific writings remaining after the death of Professor P. S. URYSOHN.

The aforesaid materials represent the latter's own scientific work and are absolutely irreplaceable. Therefore it is imperative during the border inspection to show special care for these materials and the Commissar of the Ministry of Education asks the customs officials for the possibility of an unimpeded and quick passage through the border, in any case not subjecting these materials to any risk of damage or loss.

Urysohn's legacy lives on in his works and in the topology seminar which he and Alexandroff founded during the spring semester of 1924. There is a disagreement between Pavel Sergeevich Alexandroff and Andre Nicholaevich Tychonov on the exact date of the start of the seminar: Alexandroff

recalls it as May 1924 and Tychonov remembers that it was begun at the start of the spring semester and that Alexandroff was out of Moscow at the time, joining the seminar upon his return in May.[14] The seminar has continued uninterrupted since that time except for when the university is not in session, currently meeting every Wednesday afternoon at four o'clock in Room 1403 of the main building of Moscow State University.

The seminar established by Urysohn and Alexandroff was, at least after the death of Urysohn, constructed in a form similar to previous mathematics seminars at Moscow University. The seminar of D. F. Egorov, for instance, had several groups which worked independently and then would meet in plenary sessions. However, the topology seminar stressed the independent work of the groups and did not have plenary meetings. One of the groups studied the topology of the continuum and such topics as locally connected and irreducible continua; another group had its emphasis on teaching and considered classical surfaces and curves but did not have any individual work because that was not the intent.

The central theme of a third group was abstract topology (or general topology). Among the members of this group were N. B. Vedenusov who was to work heavily with both the large and small inductive definition of dimension, and V. V. Nemytskii who, together with Stepanov, studied the

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[14] These comments were made during a conversation which was tape recorded by the author on March 1, 1979, and is referred to in the references as [CAT].

applications to topology to the qualitative theory of differential equations. However, Alexandroff considers that the first and foremost student at the time, not only chronologically as his first student of topology, but also because of the great contributions he made to the study of the subject, was Andre Nicholaevich Tychonov.

Andre Nicholaevich was born October 30, 1906, in the town of Gzhatsk in the Smolensk province. He completed secondary school by exams without having to attend classes and matriculated in Moscow State University in 1922 in the Mathematics Department of the Physical-Mathematical Faculty. In 1923 he enrolled in a special course of the topology of the continuum being given by Alexandroff. He was a diligent student in this course, taking extensive notes which he reworked every evening and thus obtained a comprehensive text for the course which he later presented to Alexandroff.

Concerning the work in the seminar, Tychonov recalls that Alexandroff would present the group with problems of constructing examples of topological spaces satisfying certain conditions, an exercise at which Andre Nicholaevich became a master and which probably was of considerable aid when he finally arrived at the construction of the product topology or, in particular, the Tychonov cube. Also presented were different problems from the Memoirs of Alexandroff and Urysohn which were to be solved independently. This method of work gave the students the self-assurance in their own abilities which enabled them to continue mathematical investigation independently.

Tychonov worked in this seminar with a great passion and often so late that he returned to his lodgings at two or three in the morning and found himself locked out because he had dawdled on the way home with his friends, speaking about mathematical problems from the seminar and unable to break off the conversation. In his memory, it was on such a walk with a friend that he discussed his lack of success on an original problem posed to him by Alexandroff: Is every normal space a subspace of a bicompact Hausdorff space? The solution to this problem became clear to him, as he remembers it, as he and his friend stood talking on the corner of Gorki Prospect and Oruzheynyy Pereulok (Weapon Lane) [CAT].

Much of the topological work of Andre Nicholaevich was done by the consideration of mappings between topological spaces and is quite obviously an offshoot of the idea of functional separation of Urysohn.[15] Indeed, it was by means of a mapping that he imbedded a completely regular (Tychonov[16]) space of weight τ in the Tychonov cube of the same weight. He felt that in mathematics it is very important to consider only the properties which are important to a problem and to eliminate the non-essential ingredients.

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[15] Two nonintersecting sets A and B are *functionally separated* in the space X if there exists a continuous function f defined on all of X for which $0 \leq f(x) \leq 1$ for arbitrary $x \in X$ and which has the value 0 for all points $x \in A$ and the value 1 for all points $x \in B$.

[16] A Tychonov or completely regular space is a T_1 space in which any point is functionally separated (see footnote 15) from any closed set which does not contain it.

It is especially important in the formulation of applied problems to be precise, but, as is often the case, the transfer of a problem from reality to mathematics inherently has some non-essential ingredients. From his work in topology he turned to the study of ill-posed problems, a work for which he would later receive the Lenin Prize.

When asked to state his reasons for leaving topology after making such great initial contributions, he answered [CAT]:

> I well understand the general scientific wealth of the results which I obtained in topology, but I wanted to build a bridge which would connect the abstract and reality. In the USSR and in the USA there are very good mathematicians and physicists, but often good mathematicians do not understand physics and conversely; and I wanted to bridge this gap.

In the opinion of Alexandroff [CAT], there are many good proofs of the Tychonov product theorem[17] with the best being that of Bourbaki, but all of these proofs are based upon the modern definition of bicompactness (by means of covers) and use such tools as the Lebesque-Borel Lemma. Andre Nicholaevich used for his first proof only the Bolzano-Weierstrass idea of complete accumulation point. In the footnotes to the third Russian edition of the Memoirs, Alexandroff noted that at the time he and Urysohn developed

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[17] In his paper "Uber die topologische Erweiterung von Raumen," Tychonov proved that the product of copies of the closed unit interval [0,1] is bicompact using, as Alexandroff points out, the concept of complete accumulation point. This proof relies heavily upon the fact that each coordinate space is a metric space. W. Rudin [Ru] observed that it was Čech who first proved that the product of bicompact spaces is bicompact and that Tychonov actually developed the "Stone-Čech compactification."

the concept of bicompactness it was done by means of complete accumulation points with secondary formulation in terms of completely ordered sequences of nonempty decreasing closed sets, and a tertiary formulation by means of open covers. Both Alexandroff and Urysohn felt that the order of the formulations represented the order of importance of each concept but, as Alexandroff pointed out, paracompactness (the most important generalization of bicompactness) is defined by means of open covers and its importance in the resolution of the general metrization theorem has shown this first impression to be incorrect.

The first results of Tychonov's work were included in a tract which Alexandroff wrote for Brouwer during the winter of 1925-26 when he was in Holland. It was also during this year that a topology circle was established at Moscow State University through the initiative of Nemytskii and Likhtenbaum. Alexandroff was elected its honorary president as the first person responsible for its direction. Members during the first year of its existence were Nemytskii, Likhtenbaum, N. B. Vedenisov, Yu. A. Roshanskaya, V. V. Stepanov, L. A. Tymarkin, A. N. Tychonov, L. A. Lysternik, L. G. Shnirel'man, A. A. Momma, and A. N. Cerkasov. The theme of the first year was the immediate continuation of the seminar of Urysohn and Alexandroff with work in three directions.

The first followed the ideas of Hausdorff and Frechet as well as Alexandroff and Urysohn on general topology. Almost every topological proof presented during that time began with the formulation of the four axioms of Hausdorff

or the three axioms of metric spaces of Frechet. The most active in this area of study were Tychonov, Vedenisov, and Nemytskii. The other two areas were geometrical in approach. One was the study of one dimensional continua and was a continuation of the work connected with the Polish mathematicians Kuratowski, Yanishevski, and Masurkevich and was studied by Likhtenbaum, Rozhanskaya, and Cherkasov. The third area of study was the idea of dimensionality based on the works of Urysohn [Ne].

With this firm footing, the topological school was established at Moscow State University, the first of such schools to evolve from the school of function theory, and the first to have its founders come from within the ranks of Luzitania, thus owing their existence to the personal magnetism and the general mathematical awareness of Nicholai Nicholaevich Luzin. This time also marked the end of Luzitania. It is generally considered that the years 1920-23 composed the Luzitania period--"the days of unforgettable Luzitania, the days of inspiration and search" [L-III]--and the years 1923-27 the post-Luzitania period.

The cause of the decline was the responsibility of Luzin as much as its rise had been. He encouraged his students to investigate mathematics and to be creative, but then would not permit deviation from his fields of interest or his direction. But, the growth and creative geniuses of his students could not be constrained within his restrictive boundaries. The school of topology and the school of the metrical theory of numbers founded by Khinchin were among the first two "defections" from Luzin's guidance. With so

much work being done by Luzin's students and other followers, some of them had to look for other avenues for their creative ambitions. But, Luzin did not understand or could not accept this and felt betrayed.

From his point of view, this betrayal reached its climax during the Academy elections in 1929. Nicholai Nicholaevich had been elected a Corresponding Member of the Academy in 1927, and looked for the support of his followers in his candidacy for full membership as an Academic in the 1929 elections. However, his candidacy was not supported by his students and, in fact, received no official support [L-III].

> Mathematics at that time was split into two large branches--"algorithmic mathematics" and "mathematics of ideas," which referred to set theory, function theory, abstract algebra, topology, etc. It was thought that only the first branch of mathematics had applications and the second existed merely "to put the mathematical house in order." In the Moscow mathematics of the twenties the mathematics of ideas ("descriptive mathematics," or "philosophy") predominated.
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> ...a well known scholar, speaking on the occasion of the elections to the Academy in support of the Candidature of N. M. Krylov for mathematics and of Luzin for philosophy showed pages from their papers, one covered with formulae, the other without formulae, and said, "This is mathematics, and this is philosophy."

The outcome of the elections was surprising for Luzin was elected an Academic in philosophy and then later, during the same election, in mathematics. Although during the last twenty years of his life he effectively isolated himself from those he considered as rebellious, his work in mathematics continued until the day of his death, and he left behind him a legacy for mathematicians not only in his own mathematical work, but in the work of his students and the schools of mathematics which they founded.

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The University of Akron
Akron, Ohio 44325