TOPOLOGY PROCEEDINGS

Volume 9, 1984

Pages 37–50

http://topology.auburn.edu/tp/

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Topology Proceedings

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ISSN: 0146-4124

PRODUCTS OF SPACES WITH ZERO-DIMENSIONAL REMAINDERS

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Abstract. A 0-*space* is a completely regular Hausdorff space possessing a compactification with zero-dimensional remainder. Let X and Y be 0-spaces. Then X × Y is a 0-space if and only if i) X and Y are locally compact, ii) X and Y are zero-dimensional or iii) one of X,Y is locally comapct and zero-dimensional. In particular, if X and Y are rimcompact, then X × Y is rimcompact if and only if X and Y satisfy i), ii) or iii).

1. Introduction

A 0-*space* is a completely regular Hausdorff space possessing a compactification with zero-dimensional remainder [2]. Recall that a space is *rimcompact* if it possesses a basis of open sets with compact boundaries. It is well-known that any rimcompact space is a 0-space and that the converse is not true (for example, see [8]). A proximal characterization of 0-spaces is presented in [2].

According to [10], if X is a rimcompact space, then X × X is rimcompact if and only if X is locally compact or X is 0-dimensional. In this paper we prove the more general result stated in the abstract.

The main results appear in Section 2. In the remainder of this section we present our notation and terminology and some known results. All spaces are assumed to be completely regular and Hausdorff. The notions used from set theory are standard. For any set X, $|X|$ denotes the cardinality of X. A *map* is a continuous surjection. The projection map from $X \times Y$ into X is denoted by π_X. A function $f: X \to Y$ is *closed* if whenever F is a closed subset of X, $f[F]$ is a closed subset of Y; f is *monotone* if $f^{\leftarrow}(y)$ is connected for each $y \in Y$. A closed function $f: X \to Y$ is *perfect* if $f^{\leftarrow}(y)$ is compact for each $y \in Y$.

The following is 9.4 of [9].

1.1 *Proposition. Let* Y *have the quotient topology induced by a map* f *from* X *onto* Y. *Then an arbitrary function* $g: Y \to Z$ *is continuous if and only if* $g \circ f: X \to Z$ *is continuous.*

A space is *zero-dimensional* (written 0-dimensional) if it has a basis of closed-and-open (denoted *clopen*) sets. The *quasicomponent* of $x \in X$ is the intersection of all clopen subsets of X containing x. The *connected component* C_x of $x \in X$ is the union of all connected subspaces of X containing x. A space X is *totally disconnected* if $C_x = \{x\}$ for each $x \in X$.

Two compactifications JX and KX of X are *equivalent* if there is a homeomorphism from JX onto KX that fixes X pointwise. We shall identify equivalent compactifications of a given space. We partially order the family $\mathcal{K}(X)$ of compactifications of X in the usual manner: $JX \leq KX$ if there is a continuous map from KX onto JX that fixes X pointwise. The maximum element of $\mathcal{K}(X)$, the *Stone-Čech compactification* of X, is denoted by βX. For background information on compactifications the reader is referred to [4] or [5].

The next result follows from 1.5 of [6].

1.2 *Proposition. Let* KX *and* KY *be compactifications of* X *and* Y *respectively, and let* f *be a perfect map from* X *onto* Y. *If there is a map* $f'\colon KX \to KY$ *such that* $f'|_X = f$, *then* $f'(KX\backslash X) = KY\backslash Y$.

We often call $KX\backslash X$ the *remainder* of KX. For any space X, the *residue* of X (denoted by R(X)) is the set of points at which X is not locally compact. If $KX \in \mathcal{K}(X)$, then $\mathrm{Cl}_{KX}(KX\backslash X) = R(X) \cup (KX\backslash X)$. The locally compact part of X will be denoted by L(X).

If A is a subset of X, the *boundary* of A in X (written $\mathrm{bd}_X A$) is the set $\mathrm{Cl}_X A \cap \mathrm{CL}_X(X\backslash A)$. If U is an open subset of X and $KX \in \mathcal{K}(X)$, the *extension* of U in KX, denoted by $\mathrm{Ex}_{KX}U$, is the set $KX\backslash \mathrm{CL}_{KX}(X\backslash U)$. It is easy to show that $\mathrm{Ex}_{KX}U$ is the largest open subset of KX whose intersection with X is the set U.

A compactification KX of X is a *perfect* compactification of X if for each open subset U of X, $\mathrm{bd}_{KX}\mathrm{Ex}_{KX}U = \mathrm{Cl}_{KX}(\mathrm{bd}_X U)$. According to the corollary to Lemma 1 of [8], βX is a perfect compactification of X. If $f\colon \beta X \to KX$ is the natural map, then KX is a perfect compactification of X if and only if f is monotone ([8]).

The following is 4.7 of [3].

1.3 *Theorem. Let* $f\colon X \to Y$ *be a monotone quotient map, and let* KX, KY *be perfect compactifications of* X, Y *respectively. If* f *extends to* g: KX → KY, *then* g *is monotone.*

A compactification whose remainder is 0-dimensional will be called *0-dimensional at infinity* (denoted by 0.I.). Any 0-space X has a maximum 0.I. compactification (which we denote by F_0X) which is also a minimum perfect compactification of X ([7]). As a consequence, if X is a 0-space, for each $p \in \beta X\backslash X$, the connected component of p in $\beta X\backslash X$ is compact and equals the quasi-component of p in $\beta X\backslash X$ (see [1] for further discussion). Let $F: \beta X \to F_0X$ be the natural map. Then for each $p \in F_0X\backslash X$, $F^{\leftarrow}(p)$ is the compact connected quasi-component (in $\beta X\backslash X$) of each element of $F^{\leftarrow}(p)$.

In [1] we introduced a natural generalization of rimcompactness called almost rimcompactness and obtained the following characterization, which we consider in this paper as a definition. A space X is *almost rimcompact* if and only if X possesses a compactification KX in which each point of KX\X has a basis (in KX) of open sets whose boundaries are contained in X. If KX is such a compactification of X, we say that KX\X is *relatively 0-dimensionally embedded* in KX. Hence each almost rimcompact space is a 0-space; in the same paper we show that the converse is not true. If X is almost rimcompact, then F_0X has relatively 0-dimensionally embedded remainder ([1]). For the internal definition and a thorough discussion of almost rimcompactness, see [1] and [3].

The following are 2.3 and 3.4 of [3] respectively.

1.4 *Proposition. Suppose* KX *is any* 0.I. *compactification of a nowhere locally compact space* X. *Then* KX *has relatively* 0-*dimensionally embedded remainder.*

1.5 *Proposition. If* F *is a closed subspace of a* 0-*space (respectively an almost rimcompact space, rimcompact space) then* F *is a* 0-*space (respectively almost rimcompact, rimcompact).*

2. The Main Results

It is well known that if X and Y are locally compact (respectively 0-dimensional) then X × Y is locally compact (respectively 0-dimensional). The following proves the sufficiency of iii) in the main result.

2.1 *Theorem. Suppose that* X *is locally compact and* 0-*dimensional and that* Y *is a* 0-*space (respectively almost rimcompact, rimcompact). Then* X × Y *is a* 0-*space (respectively almost rimcompact, rimcompact).*

Proof. Let $\omega X = X \cup \{\omega\}$ denote the one-point compactification of X. Then $\omega X \times F_0 Y$ is a compactification of $X \times Y$. Consider $K(X \times Y) = \omega X \times F_0 Y/\{\{\omega\} \times F_0 Y\}$. Clearly $K(X \times Y)$ is a compactification of $X \times Y$. Let k be the quotient map from $\omega X \times F_0 Y$ onto $K(X \times Y)$ and let $\{t\} = k[\{\omega\} \times F_0 Y]$. Now $\{\omega\} \times F_0 Y$ has a basis of clopen sets in $\omega X \times F_0 Y$, hence t has a basis of clopen sets in $K(X \times Y)$. Since ωX is 0-dimensional, if $r \in K(X \times Y)/(X \times Y)$ and $r \neq t$, then r has a basis of clopen sets in $K(X \times Y)/(X \times Y)$. Hence $K(X \times Y)$ is a 0.I. compactification of $X \times Y$.

If Y is almost rimcompact, and $(x,p) \in X \times (F_0Y\backslash Y)$, then (x,p) has a basis in $K(X \times Y)$ of open sets whose boundaries are contained in $X \times Y$. For suppose that U is a compact clopen subset of X and that V is an open subset of F_0Y such that $\mathrm{bd}_{F_0Y}V \subseteq Y$. Then $U \times V$ is open in $\omega X \times F_0Y$. Also, $\mathrm{bd}_{\omega X \times F_0Y}[U \times V] = [\mathrm{bd}_{\omega X}U \times \mathrm{Cl}_{F_0Y}V] \cup [\mathrm{Cl}_{\omega X}U \times \mathrm{bd}_{F_0Y}V] \subseteq X \times Y$. Since $k[\mathrm{Cl}_{\omega X \times F_0Y}[U \times V]] = \mathrm{Cl}_{K(X \times Y)}[U \times V]$, $\mathrm{bd}_{K(X \times Y)}[U \times V] \subseteq X \times Y$. Sets of the form $U \times V$ with the above properties form a basis in $K(X \times Y)$ for $(x,p) \in X \times (F_0Y\backslash Y)$. Since t has a basis of clopen sets in $K(X \times Y)$, $X \times Y$ is almost rimcompact.

It is easy to verify that if Y is rimcompact, then $X \times Y$ is rimcompact.

The characterization in the case where X, Y and $X \times Y$ are rimcompact will follow from the more general result for 0-spaces. It is also an easy consequence of the next result, which we feel is valuable in that it provides an internal proof of the characterization for rimcompact spaces. We do not have an internal proof of the main result for 0-spaces. (As mentioned in the introduction, an internal characterization of 0-spaces appears in [2].)

2.2 *Lemma. Suppose that* V, W *and* U *are non-empty open subsets of* X, Y *and* $X \times Y$ *respectively, and that* $(x,y) \in U \subseteq V \times W$. *If* $\mathrm{bd}_{X \times Y}U$ *is compact, and if* V *contains no clopen neighbourhoods of* x, *then* $y \in L(Y)$.

Proof. Choose V_2 and W_2 to be open in X and Y respectively such that $(x,y) \in V_2 \times W_2 \subseteq U$. We show that

$W_2 \subseteq \pi_Y[\mathrm{bd}_{X \times Y}U]$ which is compact, and therefore that $y \in L(Y)$.

Let $z \in W_2$. As $(X \times \{z\}) \cap U \subseteq V \times W$, $\pi_X[(X \times \{z\}) \cap U] \subseteq V$. Since V contains no clopen neighbourhoods of x, $\mathrm{bd}_X\pi_X[(X \times \{z\}) \cap U] \neq \phi$. Now $\pi_X\colon X \times \{z\} \to X$ is a homeomorphism, hence there is $p \in \mathrm{bd}_{X \times \{z\}}[(X \times \{z\}) \cap U]$. Then $p \in (X \times \{z\}) \cap \mathrm{Cl}_{X \times Y}U$, hence $p \in \mathrm{bd}_{X \times Y}U$. Thus $\pi_Y(p) = z \in \pi_Y[\mathrm{bd}_{X \times Y}U]$ and the statement follows.

2.3 *Theorem. Suppose that* X *and* Y *are rimcompact. Then* X × Y *is rimcompact if and only if*

i) X *and* Y *are locally compact,*

ii) X *and* Y *are* 0-*dimensional, or*

iii) X *or* Y *is locally compact and* 0-*dimensional.*

Proof. ⇐ This is obvious.

⇒ Suppose that X is not zero-dimensional. Choose $x_1 \in X$ and an open neighbourhood V_1 of x, such that V_1 contains no clopen neighbourhoods of x_1. For $y \in Y$, let W_1 be any open neighbourhood of y. As X × Y is rimcompact, there is an open subset U of X × Y with compact boundary such that $(x_1,y) \in U \subseteq V_1 \times W_1$. It then follows from 2.2 that $y \in L(Y)$. As y is an arbitrary element of Y, Y is locally compact.

If Y is not 0-dimensional, then a similar argument shows that X is locally compact.

To prove the more general result for 0-spaces we work with compactifications of X × Y rather than with X × Y itself.

2.4 *Lemma. Suppose that* X *and* Y *are* 0*-spaces and that* $f\colon X \to Y$ *is a perfect monotone map. Then there is a map* $g\colon F_\circ X \to F_\circ Y$ *such that* $g|_X = f$ *and* $g|_{F_\circ X\setminus X}\colon F_\circ X\setminus X \to F_\circ Y\setminus Y$ *is a homeomorphism.*

Proof. There is a map $F\colon \beta X \to F_\circ Y$ such that $F|_X = f$. It follows from 1.2 and 1.3 that for each $z \in F_\circ Y\setminus Y$, $F^{\leftarrow}(z)$ is a connected subset of $\beta X\setminus X$. Since $F_\circ Y\setminus Y$ is 0-dimensional, $F^{\leftarrow}(z)$ is a connected quasi-component of $\beta X\setminus X$. Let $f_\circ\colon \beta X \to F_\circ X$ denote the natural map. For each $p \in F_\circ X\setminus X$, $f_\circ^{\leftarrow}(p)$ is a connected quasi-component of $\beta X\setminus X$. Define $g\colon F_\circ X \to F_\circ Y$ as follows:

$$g(p) = f(p) \qquad \text{if } p \in X,$$
$$g(p) = F[f_\circ^{\leftarrow}(p)] \qquad \text{if } p \in F_\circ X\setminus X.$$

The map g is well-defined and $g|_X = f$. Since $g \circ f_\circ = F$, it follows from 1.1 that g is continuous and therefore closed. According to 1.2, $g[F_\circ X\setminus X] = F_\circ Y\setminus Y$. Since $g|_{F_\circ X\setminus X}$ is one-to-one, $g|_{F_\circ X\setminus X}$ is a homeomorphism.

2.5 *Lemma. Suppose that* X *is a* 0*-space and that* Y *is compact and connected. If* $X \times Y$ *is a* 0*-space, then* $F_\circ(X \times Y) \leq F_\circ X \times Y$.

Proof. The projection map $\pi_X\colon X \times Y \to X$ is a perfect monotone map. According to 2.4, there is a map $\pi_X'\colon F_\circ(X \times Y) \to F_\circ X$ such that $\pi_X'|_{X \times Y} = \pi_X$ and $\pi_X'|_{F_\circ(X \times Y)\setminus(X \times Y)}$ is a homeomorphism. Let $\pi_\circ$ denote the projection map from $F_\circ X \times Y$ into $F_\circ X$. Define $g\colon F_\circ X \times Y \to F_\circ(X \times Y)$ as follows:

$$g((p,q)) = (p,q) \qquad \text{if } (p,q) \in X \times Y,$$
$$g((p,q)) = \pi_X'^{\leftarrow}(p) \qquad \text{if } (p,q) \in (F_\circ X\setminus X) \times Y.$$

Clearly $\pi_X' \circ g = \pi_\circ$, and $g|_{X \times Y}$ is the identity map. Let $\beta\colon \beta(X \times Y) \to F_\circ(X \times Y)$ and $F\colon \beta(X \times Y) \to F_\circ X \times Y$ denote the natural maps. According to 1.1, to show that g is continuous it suffices to show that $g \circ f = \beta$. Clearly $(g \circ F)|_{X \times Y} = \beta|_{X \times Y}$. Suppose that $t \in \beta(X \times Y)\setminus(X \times Y)$ and that $(g \circ F)(t) \neq \beta(t)$. Since $\pi_X'|_{F_\circ(X \times Y)\setminus(X \times Y)}$ is one-to-one, $(\pi_X' \circ g \circ F)(t) \neq (\pi_X' \circ \beta)(t)$. However, $\pi_X' \circ g = \pi_\circ$, hence $\pi_\circ \circ F \neq \pi_X' \circ \beta$. As this is a contradiction, g is continuous.

2.6 *Theorem. Suppose that* $X \times Y$ *is a* 0-*space. If* Y *contains a non-degenerate compact connected subset then* X *is locally compact. In particular, if* Y *is locally compact and not* 0-*dimensional, then* X *is locally compact.*

Proof. We show first that if Y is compact, connected and $|Y| > 1$, then X is locally compact. According to 2.5, there is a map $g\colon F_\circ X \times Y \to F_\circ(X \times Y)$ such that g preserves $X \times Y$ pointwise. Since Y is connected, and $F_\circ(X \times Y)\setminus(X \times Y)$ is 0-dimensional, $|g[\{q\} \times Y]| = 1$ for each $q \in F_\circ X\setminus X$. It is easy to see that if $p \in R(X) = [\mathrm{Cl}_{F_\circ X}(F_\circ X\setminus X)] \cap X$ and $y_1 \neq y_2 \in Y$, then (p,y_1) and (p,y_2) cannot have disjoint neighbourhoods in $F_\circ(X \times Y)$. Thus $R(X) = \phi$.

Now suppose that Y contains any non-degenerate compact connected subset C. Then $X \times C$ is a closed subspace of a 0-space, hence $X \times C$ is a 0-space. It follows from the argument in the preceding paragraph that X is locally compact.

It is easy to verify that if Y is locally compact, and compact connected subsets of Y consist of at most one point, then Y is 0-dimensional. The theorem is proved.

We point out that a space which contains only degenerate compact connected subsets may be connected. (See, for example, 29.2 of [9].) Also, a totally disconnected 0-space need not be 0-dimensional, even if the space is rimcompact (4.7 of [1]).

The next result will be instrumental in strengthening 2.6 to conclude that if Y is not 0-dimensional, then X is locally compact.

2.7 *Lemma. Suppose that* K *is a compactification of* $X \times Y$ *with relatively* 0*-dimensionally embedded remainder, and that* $\mathrm{Cl}_K[X \times \{y\}]$ *is connected for each* $y \in Y$. *Then* X *is locally compact or* Y *is locally compact.*

Proof. It suffices to show that if $(x,y) \in X \times Y$, then $x \in L(X)$ or $y \in L(Y)$.

For $(x,y) \in X \times Y$, choose U_1 and V_1 to be proper open neighbourhoods of x and y respectively. There is an open set T of $X \times Y$ such that $(x,y) \in T \subseteq \mathrm{Cl}_K T \subseteq \mathrm{Ex}_K(U_1 \times V_1)$. Since K has relatively 0-dimensionally embedded remainder, for each $p \in \mathrm{Cl}_K T \setminus (X \times Y)$ there is an open set U_p of K such that $p \in U_p \subseteq \mathrm{Cl}_K U_p \subseteq \mathrm{Ex}_K(U_1 \times V_1)$ and $\mathrm{bd}_K U_p \subseteq X \times Y$.

Then $\mathrm{Cl}_K T \setminus \cup\{U_p\colon p \in \mathrm{Cl}_K T \setminus (X \times Y)\}$ is a compact subset R of $X \times Y$. Suppose that $T \cap (X \times \{y\}) \subseteq R$. Then $\pi_X[T \cap (X \times \{y\})] \subseteq \pi_X[R]$ which is compact, hence $x \in L(X)$.

On the other hand, if $T \cap (X \times \{y\}) \nsubseteq R$, then there is $x' \in X$ such that $(x',y) \in T \cap U_p$ for some $p \in \mathrm{Cl}_K T \setminus (X \times Y)$. Choose U_2 and V_2 to be open neighbourhoods of x' and y respectively such that $U_2 \times V_2 \subseteq U_p \cap (X \times Y)$. We show that $V_2 \subseteq \pi_Y[\mathrm{bd}_K U_p]$ and therefore that $y \in L(Y)$.

If $z \in V_2$, then $[\mathrm{Cl}_K(X \times \{z\})] \cap U_p$ is a non-empty open subset of $\mathrm{Cl}_K(X \times \{z\})$. Since $U_p \cap (X \times \{z\}) \subseteq U_1 \times \{z\} \neq X \times \{z\}$, $[\mathrm{Cl}_K(X \times \{z\})] \cap U_p \neq \mathrm{Cl}_K(X \times \{z\})$. As $\mathrm{Cl}_K(X \times \{z\})$ is connected, it follows that the boundary in $\mathrm{Cl}_K(X \cap \{z\})$ of $[\mathrm{Cl}_K(X \times \{z\})] \cap U_p$ is non-empty. If p is any element of this boundary, then $p \in [\mathrm{Cl}_K(X \cap \{z\})] \cap [\mathrm{Cl}_K U_p]$. It follows that $p \in \mathrm{bd}_K U_p \subseteq X \times Y$. Hence $p \in X \times \{z\}$, and $\pi_Y(p) = z \in \pi_Y[\mathrm{bd}_K U_p]$. Thus $V_2 \subseteq \pi_Y[\mathrm{bd}_K U_p]$ and $y \in L(Y)$.

We now prove the main result.

2.8 *Theorem. Let* X *and* Y *be* 0-*spaces. Then* X × Y *is a* 0-*space if and only if*

i) X *and* Y *are locally compact,*

ii) X *and* Y *are* 0-*dimensional, or*

iii) X *or* Y *is locally compact and* 0-*dimensional.*

Proof. ⇐ This follows from 2.1 and the fact that the properties of 0-dimensionality and local compactness are productive.

⇒ It suffices to show that if X is not 0-dimensional, then i) or iii) holds.

Suppose that X contains non-degenerate compact connected subsets. According to 2.6, Y is locally compact. It also follows from 2.6 that if Y is not 0-dimensional, then X is locally compact.

Suppose then that X contains only degenerate compact connected subsets but that X is not 0-dimensional. Note that this implies that X is not locally compact. Either X is totally disconnected or X contains some non-degenerate connected subset. We show that in either case, X contains a closed nowhere locally compact subset C such that $\mathrm{Cl}_{F_\circ X}C$ is connected.

If X contains a non-degenerate closed connected subset C, then $\mathrm{Cl}_{F_\circ X}C$ is connected. Suppose that $\mathrm{Cl}_{F_\circ X}[\mathrm{Cl}_{F_\circ X}C \cap (F_\circ X\backslash X)] \neq \mathrm{Cl}_{F_\circ X}C$. Then $C' = \mathrm{Cl}_{F_\circ X}C\backslash\mathrm{Cl}_{F_\circ X}[\mathrm{Cl}_{F_\circ X}C \cap (F_\circ X\backslash X)]$ is an open locally compact subset of C. Since X, and therefore C, contains only degenerate compact connected subsets, C' is 0-dimensional. However, C is connected, hence cannot contain any open 0-dimensional subspaces. This contradiction implies that $C' = \phi$. That is, C is nowhere locally compact.

If X is totally disconnected but not 0-dimensional, then $F_\circ X$ is not 0-dimensional. Choose K to be any compact connected subset of $F_\circ X$ such that $|K| > 1$. It follows from an argument similar to that in the preceding paragraph that $K \cap X$ is nowhere locally compact and that $\mathrm{Cl}_{F_\circ X}(K \cap X) = K$.

Thus if X contains only degenerate compact connected subsets but is not 0-dimensional, there is a closed nowhere locally compact subset C of X such that $\mathrm{Cl}_{F_\circ X}C$ is connected. Then $\mathrm{Cl}_{F_\circ(X \times Y)}[C \times Y]$ is a 0.I. compactification of the closed subspace $C \times Y$ of $X \times Y$. As $C \times Y$ is nowhere locally compact, it follows from 1.4 that $\mathrm{Cl}_{F_\circ(X \times Y)}[C \times Y]$ has relatively 0-dimensionally embedded remainder. We claim that $\mathrm{Cl}_{F_\circ(X \times Y)}[C \times \{y\}]$ is connected for each $y \in Y$. For $\mathrm{Cl}_{F_\circ(X \times Y)}[C \times \{y\}] \subseteq \mathrm{Cl}_{F_\circ(X \times Y)}[X \times \{y\}]$ which is a 0.I. compactification of $X \times \{y\}$. If $g_y\colon F_\circ X \to \mathrm{Cl}_{F_\circ(X \times Y)}[X \times \{y\}]$ is the natural map, then $g_y[\mathrm{Cl}_{F_\circ X}C] = \mathrm{Cl}_{F_\circ(X \times Y)}[C \times \{y\}]$. Thus the latter is connected and the claim is proved.

It follows from 2.7 that C or Y is locally compact. Since C is nowhere locally compact, Y is locally compact. Since X is not locally compact, 2.6 implies that Y is also 0-dimensional. The theorem is proved.

We point out that it is an immediate consequence of 2.8 that no compactification of $Q \times I$ has 0-dimensional remainder (where Q, I denote the rational numbers and unit interval, respectively).

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