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PROBLEM SECTION

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PROBLEM SECTION

CONTRIBUTED PROBLEMS

Most of the problems listed here are related to talks at the 1987 Spring Topology Conference in Birmingham, Alabama, for which this volume of TOPOLOGY PROCEEDINGS is the journal of record.

Anyone who has published a paper in TOPOLOGY PROCEEDINGS or attended a Spring Topology Conference is invited to submit problems to this section. They need not be related to any articles, but if they are, references are welcome. Terms not in a general topology text or in referenced articles should be defined.

B. Generalized Metric Spaces and Metrization

31. *(C. R. Borges)* If $(X,\mathcal{J})$ is a topologically complete submetrizable topological space, is there a complete metric for X whose topology is coarser than $\mathcal{J}$?

32. *(Nyikos)* Is it consistent that every compact space with hereditarily collectionwise Hausdorff (cwH) square is metrizable? [*Note*. If MA + $\neg$CH, then every compact space with hereditarily strongly cwH square is metrizable, but this is false under CH.]

33. *(Nyikos)* Can the consistency of "all normal Moore spaces of cardinality $\leq \kappa$ are metrizable" be established without using large cardinals if (a) $\kappa = c$, (b) $\kappa = 2^{c}$, (c) $\kappa < \beth_\omega$?

C. Compactness and Generalizations

55. *(Nyikos)* Is $2^{\mathfrak{s}}$ always the smallest cardinalty of an infinite compact Hausdorff space with no nontrivial convergent sequences? [Here $\mathfrak{s}$ denotes the splitting number, which can be characterized as the least cardinal κ such that 2^{κ} is not sequentially compact. Fedorchuk has shown, in effect, that there is always a compact Hausdorff space of cardinality $2^{\mathfrak{s}}$ with no nontrivial convergent sequences.]

56. *(Nyikos)* Is it consistent that every separable, hereditarily normal, countably compact space is compact?

See also B32 and H16.

F. Continua Theory

30. *(J. Grispolakis)* If Y is an LC' continuum with no local separating points does (Y, y_0) have the avoidable arcs property for some $y_0 \in Y$?

See also the following three problems.

G. Mappings of Continua and Euclidean Spaces

20. *(J. Grispolakis)* Let $f: X \to Y$ be a weakly confluent mapping from a compact connected PL n-manifold X onto a PL m-manifold Y with $n, m \geq 3$. Is f homotopic to a light open mapping of X onto Y?

21. *(J. Grispolakis)* Let $f: M \to Y$ be a mapping from a compact connected PL n-manifold, $n \geq 3$, into an ANR Y such that every simple closed curve can be approximated by a spiral in Y. If $\pi(Y)$ has property (Tor) relative to $f_{\#}\pi(M)$, is f homotopic to a weakly confluent mapping of M onto Y?

22. *(J. Grispolakis)* Characterize all weakly confluent images of the 3-cube.

H. Homogeneity and Mappings of General Spaces

15. *(C. R. Borges)* Let θ be a family of gages for a set X, θ^{**} the gage for X generated by θ. If $f: X \to X$ is (θ,ξ)-expansive for some $\xi > 0$, is f also (θ^{**},ξ)-expansive?

16. *(C. R. Borges)* Let (X,U) be a sequentially compact (or countably compact or pseudocompact) uniform space and θ a subgage for U. If $f: X \to X$ is a continuous (wrt the uniform topology) (θ,ξ_0)-expansive map for some $\xi_0 > 0$, is $f(X) = X$?

See also P29 and P30.

P. Products, Hyperspaces, Remainders, and Similar Constructions

29. *(B. Lawrence)* Let $X = \square^{\omega}\mathbb{Q}$, $Y = \nabla^{\omega}\mathbb{Q}$ and $\sigma: X \to Y$ the natural quotient map. [Recall that two points of X have the same image iff they disagree on at most finitely many coordinates.] Is there a closed subset C of X such that $\sigma[C]$ is dense in Y and C contains at most one point in each fiber?

30. *(B. Lawrence)* Replace the rationals by the irrationals in P29.

U. Uniform Spaces

3. *(C. R. Borges)* If (X,U) is topologically complete, is there a subgage θ for U such that each $\rho \in \theta$ is a complete pseudometric?

See also B31, H15, and H16.

Z. Topological Dynamics, Fractals and Hausdorff Dimension

1. *(P. Massopust)* What is the fractal dimension of G = graph (f) when f is a fractal interpolation function generated by polynomials or general C^0-maps? Is it possible to calculate the fractal dimension in this case by an approximation scheme consisting of affine and/or polynomial maps?

2. *(P. Massopust)* What are the fractal dimensions of $A(I \times X)$ and graph (f*), when f* is a hidden variable fractal interpolation function generated by affine, or even more general C^0-maps, rather than by similitudes? Is it still true that $\dim A(I \times X) = \dim(X)$, or under what condition(s) does this relation remain valid?

3. *(P. Massopust)* What is the exact Hausdorff-Besicovitch dimension for the graph of a fractal interpolation and hidden variable interpolation function?

INFORMAION ON EARLIER PROBLEMS

From Arkhangel' skii's Survey Paper

[Uspehii Mat. Nauk. 33:6 (1978) = Russ. Math. Surv. 33:6, 33-96]

8. Does there exist "naïvely" a regular space for which $hL(X^n) \leq \tau$ for all $n \in \mathbb{N}$ and $d(X) > \tau$? *Solutions for various* τ. Yes if τ is a (singular) cardinal satisfying $2^{cf\tau} < \tau$ and $\tau^{<cf\tau} = \tau$, hence yes for all singular strong limit cardinals. (Todorčević, Compositio Math. 57 (1986) Theorem 16, Lemmas 17 through 19, and the comment in the proof of Lemma 18.) This partially complements, and predates, Shelah's result ("Yes for any τ that is the

successor of a regular cardinal except possibly $\tau = \omega_2$")
announced last year.

9. Does there exist "naïvely" a ccc non-separable compact Hausdorff space of cardinality $\leq 2^{\aleph_0}$? *Solution.* Yes. (Todorčević and Veličković, Compositio Math. 63 (1987), 391-408).

From TOPOLOGY PROCEEDINGS

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C16. *(J. Hagler)* Does there exist a compact space K with a countable dense set D such that every sequence from D has a convergent subsequence, but K is not sequentially compact? *Solution.* Yes. *(A. Dow)*.

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C51. *(Malyhin)* A space is called weakly first countable if to each point x one can assign a countable filterbase $\mathcal{F}_x$ of sets containing x wuch that a set U is open iff for each $x \in U$ there is $F \in \mathcal{F}_x$ such that $F \subset U$. Is there a weakly first countable compact space which is not first countable? One that is of cardinality $> \mathfrak{c}$? *Consistency results.* Yes to the first question if $\mathfrak{b} = \mathfrak{c}$ *(H. Zhou)*. If $\aleph_1$ dominating reals are iteratively added and every countable subset of ω appears at some initial stage, then "arbitrarily large" weakly first countable compact Hausdorff spaces exist *(Nyikos)*.

C54. *(Nyikos)* Is there a compact sequential space of nonmeasurable cardinality that is not hereditarily α-realcompact? *Solution.* Yes *(A. Dow)*.

F29. *(Nyikos)* Is there a (preferably first countable or, better yet, perfectly normal) locally connected continuum without a base of open sets with locally connected closures? *Partial solution.* *(Gruenhage)* There is a first countable example. If CH is assumed, there is one which is perfectly normal as well.

P27. *(Malyhin)* A point $x \in X$ is called a butterfly point if there exist disjoint sets A and B of X such that $\overline{A} \cap \overline{B} = \{x\}$. Is it consistent that there is a non-butterfly point in $\omega^* = \beta\omega - \omega$? Is it consistent with MA? *Partial solution.* *(Beslagic and van Douwen)* It is not consistent with MA. In fact, if $\mathfrak{r} = \mathfrak{c}$, then every point of ω^* is a butterfly point; more strongly, $\omega^* - \{p\}$ is non-normal for every $p \in \omega^*$. Here $\mathfrak{r}$ denotes the reaping number, i.e. the least κ for which there is a family $\mathcal{R}$ of κ subsets of ω such that if A is a subset of ω, then A does not reap $\mathcal{R}$, i.e. there is $R \in \mathcal{R}$ such that either $R \subset^* A$ or $R \cap A$ is finite.