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PRODUCTS OF PARACOMPACT SPACES AND LAŠNEV SPACES

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Dedicated to Professor Yukihiro Kodama on his 60th birthday

ABSTRACT. We shall prove that for a Lašnev space Y the product $X \times Y$ is paracompact for any paracompact space X iff Y is σ-locally compact. This extends K. Morita's theorem for the metrizable case.

1. INTRODUCTION.

A space which is the closed image of a metric space was characterized by Lašnev [12], and it is called a Lašnev space. The normality of a product space $X \times Y$ with a Lašnev space Y has been studied by Chiba [4], Bešlagić and Chiba [3] and the author [7], [8]. Their results generalize or closely relate to those obtained so far for the case Y metrizable.

Spaces Y with the property that $X \times Y$ is paracompact for any paracompact space X have been long considered by many authors (for surveys, see Przymusiński [16], Atsuji [2]), but they are not yet characterized explicitly (cf. Katuta [10]). Following Telgárski [18] let us denote by $\prod$ the class of such Y's. In case Y is metrizable Morita [14] proved a theorem that $Y \in \prod$ iff Y is σ-locally compact (= a countable union of closed locally compact subspaces); the "if" part is always true for an arbitrary paracompact space Y (Morita [15]). This theorem of Morita is also valid for a paracompact M-space Y (Morita[15]), but is not valid when Y is the closed image of a paracompact M-space; indeed, in [9] Ishii constructed a space which is the closed image of a locally compact paracompact

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space, and hence belong to $\prod$ (see[18]), but is not σ-locally compact. In view of these facts it is natural to ask for what spaces Y the theorem of Morita above is true, especially for Lašnev spaces. In this paper we will show it is true for Lašnev spaces; that is, we establish a theorem that a Lašnev space Y belongs to $\prod$ iff it is σ-locally compact.

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2. Proof of the Main Theorem

All spaces considered in this paper are assumed to be Hausdorff and closed maps continuous onto. N denotes the set of positive integers.

The following lemma is fundamental in our discussions.

Lemma 2.1. *Let Y be a Lašnev space. Then there exist a metric space T and a one-to-one onto continuous map $f : Y \to T$ such that $T - f(D_0)$ is σ-discrete (= a countable union of closed discrete subsets of T), where*

$$D_0 = \{y \in Y | \ \{f^{-1}(G) | G \text{ is an open nbd of } f(y)\} \text{ is a nbd base of } y \text{ in } Y\}.$$

Proof. This is essentially proved in Arhangel'skiǐ [1]. Indeed, let Z be a metric space and $g : Z \to Y$ a closed map. Then by Lašnev's decomposition theorem (Lašnev [11]) $Y = Y_0 \cup \cup_{i \in N} Y_i$, where for each $y \in Y_0$ $g^{-1}(y)$ is compact, each Y_i, $i \in N$ is closed discrete, and $Y_0 \cap \cup_{i \in N} Y_i = \emptyset$. Then in the proof of his theorem Arhangel'skiǐ [1] constructed a metric space T and a one-to-one onto continuous map $f : Y \to T$ such that each point $y \in Y_0$ has $\{f^{-1}(G) \mid G \text{ is an open nbd of } f(y) \text{ in } T\}$ as its nbd base in the whole space Y and each $f(Y_i)$, $i \in N$ is closed discrete in T. Hence, if we define D_0 as above, we have $Y_0 \subset D_0$ and $T - f(D_0) \subset \cup_{i \in N} f(Y_i)$. Hence $T - f(D_0)$ is σ-discrete.

In what will follow whenever we mention a Lašnev space $Y, T, f : Y \to T$ and D_0 are defined as in Lemma 2.1, and let

$T_0 = f(D_0)$, $T - T_0 = \cup_{i \in N} T_i$, where T_i, $i \in N$ is closed discrete in T, and $D_i = f^{-1}(T_i)$, $i \in N$.

We shall.prove some preliminary results. First we show

Theorem 2.2. *Let X be a countably paracompact normal space and Y a Lašnev space. If $X \times Y$ is normal, then $X \times T$ is normal.*

Proof. Let E and F be disjoint closed subsets of $X \times T$. Let $i \geq 1$ and let t be any point of T_i. Since $X \times \{t\}$ is normal, there exists an open set G_i of X such that

$$E \cap (X \times \{t\}) \subset G_t \times \{t\} \text{ and } (\overline{G}_t \times \{t\}) \cap F = \emptyset.$$

Then we have $((\overline{G_t} \times T) \cap F) \cap (X \times \{t\}) = \emptyset$. Since X is countably paracompact normal and t has a countable nbd base, it is easy to see that there exists an open set H_t of $X \times T$ such that

$$X \times \{t\} \subset H_t \text{ and } \overline{H}_t \cap ((\overline{G}_t \times T) \cap F)) = \emptyset.$$

Since T_i is closed discrete and T is metrizable, there exists an open nbd W_t of t for each $t \in T_i$ such that $\{W_t \mid t \in T_i\}$ is discrete. Let

$$U_i = \cup\{(G_t \times Y) \cap H_t \cap (X \times W_t) \mid t \in T_i\}$$

Then U_i is an open set of $X \times T, E \cap (X \times T_i) \subset U_i$ and $\overline{U}_i \cap F = \emptyset$. Let $E_0 = E - \cup_{i \in N} U_i$. Then E_0 is closed and contained in $X \times T_0$. Let $1_X \times f : X \times Y \to X \times T$ be the product map, where 1_X is the identity on X. Since $X \times Y$ is normal, there eixsts an open set G of $X \times Y$ such that

$$(1_X \times f)^{-1}(E_0) \subset G \text{ and } \overline{G} \cap (1_X \times f)^{-1}(F) = \emptyset.$$

Now let $\mathcal{B} = \cup_{i \in N} \mathcal{B}_i$ be a σ-locally finite base for T, where $\mathcal{B}_i$ is locally finite. For each $B \in \mathcal{B}_i$ define an open set G_B of X by

$$G_B = \cup\{P \mid P \text{ is open in } X \text{ such that } P \times f^{-1}(\overline{B}) \subset G\}.$$

Since $(1_X \times f)^{-1}(E_0) \subset X \times D_0$, and by the definition of D_0, we have

$$(1_X \times f)^{-1}(E_0) \subset \cup\{G_B \times f^{-1}(\overline{B}) \mid B \in \mathcal{B}\}.$$

Moreover, since $G_B \times f^{-1}(\overline{B}) \subset G$ and $\overline{G} \cap (1_X \times f)^{-1}(F) = \emptyset$, we have

$$(\overline{G}_B \times \overline{B}) \cap F = \emptyset.$$

Let $V_i = \cup\{G_B \times B \mid B \in \mathcal{B}_i\}$. Then $\overline{V}_i \cap F = \emptyset$ since $\mathcal{B}_i$ is locally finite. Since we have $E_0 \subset \cup_{i \in N} V_i$, the above shows that U_i and V_i, $i \in N$ are open sets of $X \times T$ which satisfy

$$E \subset \cup_{i \in N} U_i \cup \cup_{i \in N} V_i,\ \overline{U}_i \cap F = \overline{V}_i \cap F = \emptyset,\ i \in N.$$

Similarly we can find open sets U_i' and V_i' of $X \times T$ so that $F \cap (X \times T_i) \subset U_i', \overline{U}_i' \cap E = \emptyset$ for each $i \in N$and $F - \cup_{i \in N} U_i' \subset \cup_{i \in N} V_i', \overline{V}_i' \cap E = \emptyset$ for each $i \in N$. That is,

$$F - \cup_{i \in N} U_i' \cup \cup_{i \in N} V_i', \overline{U}_i' \cap E = \overline{V}_i' \cap E = \emptyset, i \in N.$$

This shows, as is well-known, E and F are separated by open sets. Hence $X \times T$ is normal

Remark. The converse of this theorem does not hold. For let X be the Micheal line. Let Y be the space obtained from the real line R by making each rational number isolated (Michael [13]). Then Y is metrizable. Clearly the identity map $f : Y \to R$ satisfies the properties in Lemma 2.1. $X \times R$ is paracompact, but $X \times Y$ is not normal.

Corollary 2.3. *Let Y be a Lašnev space. If Y belongs to $\prod$, then T is σ-locally compact.*

Proof. Let X be a paracompact space. Then $X \times Y$ is normal. Hence by Theorem 2.2 $X \times T$ is normal. Therefore, by Rudin-Starbird [17] and Morita [14] $X \times T$ is paracompact. That is, T belongs to $\prod$. Hence by Morita [14], T is σ-locally compact.

Let $\beta f : \beta Y \to \beta T$ be the Stone extension of $f : Y \to T$.

Lemma 2.4. *$\beta f^{-1}(f(y)) \cap Y = \{y\}$ for any $y \in \cup_{i \in N} D_i$. $\beta f^{-1}(f(y)) = \{y\}$ for any $y \in D_0$. Hence $\beta f^{-1}(T) = \cup_{i \in N} \beta f^{-1}(T_i) \cup D_0$.*

Proof. Since f is one-to-one, the first statement is obvious. To prove the second, let $y \in D_0$. Suppose there is $y' \in \beta f^{-1}(f(y))$ with $y' \neq y$. Since $y \in D_0$, there exist an open nbd G of $f(y)$ and a nbd H of y' in βY such that $f^{-1}(G) \cap H = \emptyset$. Since $\beta f(y') = f(y)$, we may assume futher $\beta f(H) \cap T \subset G$. Take $y'' \in H \cap Y$. Then $y'' \in f^{-1}(G) \cap H$, a contradiction.

Lemma 2.5. *For each $y \in \cup_{i \in N} D_i$ there exists an open nbd V_y of y such that $f^{-1}(G) - \overline{V}_y \neq \emptyset$ for any open nbd G of $f(y)$ in T.*

Proof. The lemma follows since Y is regular and $y \notin D_0$.

Let $y \in \cup_{i \in N} D_i$ and V_y be as in Lemma 2.5. Define

$$C_y = \beta f^{-1}(f(y)) - V_y^*,$$

where $V_y^* = \beta Y - \overline{Y - V_y}$. Then using Lemma 2.5 and the fact that βf is a closed map, it is easy to see that C_y is non-empty. Pick a point $p_y \in C_y$ for each $y \in \cup_{i \in N} D_i$. Define a subspace Y_c of βY by

$$Y_c = \{p_y \mid y \in \cup_{i \in N} D_i\} \cup D_0.$$

We shall prove the following lemma.

Lemma 2.6. *Y_c is a hereditarily paracompact space.*

Proof. We shall only prove Y_c is paracompact. The heredity can be proved similarly. Clearly Y_c is a regular space. Let $g = \beta f \mid Y_c : Y_c \to T$. Let $\mathcal{B}$ be a σ-locally finite base of T. Let $\mathcal{U}$ be an open cover of Y_c. Define $\mathcal{V}_0 =$
$\{g^{-1}(B) \mid B \in \mathcal{B}, g^{-1}(B)$ is contained in some member of $\mathcal{U}\}$.

$\mathcal{V}_0$ is a σ-locally finite collection of open sets of Y_c and it is easy to see $\cup \mathcal{V}_0 \supset D_0$. Let $i \in N$. Since $f(D_i) = T_i$ and T_i is closed discrete in T, there exists an open nbd $W_{f(y)}$ of $f(y)$ for each $y \in D_i$ such that $\{W_{f(y)} \mid y \in D_i\}$ is discrete in T.

Since $g(p_y) = f(y)$, we can take an open nbd V_{p_y} of p_y in Y_c such that $g(V_{p_Y}) \subset W_{f(y)}$ and V_{p_y} contained in some member of $\mathcal{U}$. Define $\mathcal{V}_i = \{V_{p_y} \mid y \in D_i\}$. Now the above show $\cup_{0 \leq i}\mathcal{V}_i$ is a σ-locally finite open cover of Y_c which refines $\mathcal{U}$. Hence Y_c is paracompact.

Remark. Y_c is, in addition, a σ-space. For,

$$\{\{p_y\} | y \in D_i, i \in N\} \cup \{g^{-1}(B) \mid B \in \mathcal{B}\}$$

is a σ-locally finite net for Y_c.

Let us define a space M_Y. Let M_Y be the space obtained from the set Y_c with the topology generated by $\{U \mid U$ open in Y_c or $U \subset D_0\}$, hence points in D_0 are isolated.

Since by Lemma 2.6 Y_c is hereditarily paracompact, M_Y is also hereditarily paracompact (see [5]).

Modifying Michael's proof that Michael line has a non-normal product with the irrationals [13], we show:

Lemma 2.7. *Let Y be a Lašnev space. Assume that the metric space T is Čech-complete. If Y belongs to $\prod$ then either D_0 or $\cup_{i \in N} D_i$ is not dense in Y.*

Proof. Suppose D_0 is dense in Y. We show $\cup_{i \in N} D_i$ is not dense in Y. Let Y_c and M_Y be spaces defined above. Let $E = \{p_y \mid y \in \cup_{i \in N} D_i\}$. Consider the product $M_Y \times Y$. Define

$$A = E \times Y \text{ and } B = \{(y, y) \mid y \in D_0\}.$$

Then $A \cap B = \emptyset$. Clearly A is closed in $M_Y \times Y$. Since $M_Y \cap Y = D_0, B$ is closed in $M_Y \times Y$. Since $M_Y \times Y$ is normal, there exist open sets K and L in $M_Y \times Y$ such that

$$A \subset K, B \subset L \text{ and } K \cap L = \emptyset.$$

Consider the subspace $M_Y \times D_0$, and let

$$A' = E \times D_0,\ K' = K \cap (M_Y \times D_0), L' = L \cap (M_Y \times D_0).$$

Then $A' \subset K', B \subset L'$ and $K' \cap L' = \emptyset$.

Let d be a metric on T which defines the topology of T. Let $g = \beta f \mid Y_c : Y_c \to T$. Since D_0 is a subspace of Y_c, by

Lemma 2.4 for each $y \in D_0$ there exist an $n \in N$, such that $\{y\} \times (g^{-1}(B(g(y), \frac{1}{n})) \cap D_0) \subset L'$, where $B(*, \epsilon)$ is the usual ϵ-ball. For $n \in N$, define

$$P_n = \{y \in D_0 \mid \{y\} \times (g^{-1}(B(g(y), \frac{1}{n})) \cap D_0) \subset L'\}.$$

Then $D_0 = \cup_{n \in N} P_n$. We shall show $D_0 = \cup_{n \in N} \overline{P}_n^{Y_c}$, where $\overline{P}_n^{Y_c}$ is the closure of P_n in Y_c . Suppose not. Then for some $n \in N, E \cap \overline{P}_n^{Y_c}$ contains a point p_y. Take $0 < \epsilon < \frac{1}{2n}$ and choose $y' \in g^{-1}(B(g(p_y), \epsilon)) \cap P_n$. Since $(p_y, y') \in A' \subset K'$, there exists a nbd U of p_y in M_Y such that $U \times \{y'\} \subset K$. Since U is a nbd of p_y also in Y_c, we may assume $U \subset g^{-1}(B(g(p_y), \epsilon))$. Since $p_y \in \overline{P}_n^{Y_c}, U \cap P_n \neq \emptyset$. Choose $y'' \in U \cap P_n$. Now the above shows that $d(g(y''), g(y')) < \frac{1}{n}$ and $\{y''\} \times (g^{-1}(B(g(y'')), \frac{1}{n})) \cap D_0) \subset L'$. Hence $(y'', y') \in L'$, whereas $(y'', y') \in U \times \{y'\} \subset K'$, which contradicts $K' \cap L' = \emptyset$. Thus $D_0 = \cup_{n \in N} \overline{P}_n^{Y_c}$. Observe that D_0 is Čech-complete since D_0 is homeomorphic to $f(D_0) = T_0$ and $T_0 = T - \cup_{i \in N} T_i$ is a G_δ-subset of Čech-complete T. And note that D_0 is dense in Y_c since D_0 is dense in Y. Thus, by Baire's theorem for some $n \in N$ we have

$$\emptyset \neq \mathrm{Int}_{Y_c} \overline{P}_n^{Y_c} \subset \overline{P}_n^{Y_c} \subset D_0.$$

This shows E is not dense in Y_c . By definition of D_0 this implies that $\cup_{i \in N} D_i$ is not dense in Y.

We shall now prove our main theorem.

Theorem 2.8. *Let Y be a Lašnev space. Then $X \times Y$ is paracompact for any paracompact space X iff Y is σ-locally compact.*

Proof. We only prove the "only if" part. Assume that $X \times Y$ is paracompact for any paracompact space X, that is, Y belongs to $\prod$. Then, by Corollary 2.3 T is σ-locally compact, or equivalently, T has a σ-locally finite cover $\{C_\lambda\}$ of compact subsets. Let $Y_\lambda = f^{-1}(C_\lambda)$ and $f_\lambda = f \mid Y_\lambda : Y_\lambda \to C_\lambda$. Then Y_λ is a Lašnev space and closed in Y, and hence $Y_\lambda \in \prod$. Therefore, if we could prove that Y_λ is σ-locally compact, or equivalently, Y_λ has a σ-locally finite cover of compact subsets, then, since $\{Y_\lambda\}$ is a σ-locally finite cover of closed subsets, Y would have a σ-locally finite cover of compact subsets, this is Y would be σ-locally compact. Note that Y_λ, C_λ and $f_\lambda : Y_\lambda \to C_\lambda$ have the same properties as those for $Y, T, f : Y \to T$ described in Lemma 2.1. Thus, without loss of generality, we may and shall assume that T is compact.

First we note that Y is hereditarily Lindelöf . For, D_0 is homeomorphic to T_0 which is separable metrizable, and each $T_i (i \geq 1)$ is a finite set since it is closed discrete in T.

For a subspace A of Y we define

$$A_0 = \{y \in A \mid \ \{f^{-1}(G) \cap A \mid G \text{ is an open nbd of } f(y)\} \text{ is a nbd base of } y \text{ in } A\},$$

and $A_1 = A - A_0$. Hence Y_0 and Y_1 are D_0 and $\cup_{i \in N} D_i$ used above, respectively. Note that $Y_0 \cap A \subset A_0$ and $\mid A_1 \mid \leq \omega$.

We shall prove a claim

Claim. *Let F be a closed subset of Y . Then $f(\overline{F}_0 \cap \overline{F}_1)$ is G_σ in $f(F)$.*

Proof of Claim. Observe that $f(\overline{F}_1)$ is closed in $f(F)$, and it is G_δ in $f(F)$. Since $f(F) - f(\overline{F}_0) = f(F - \overline{F}_0) \subset f(F_1)$ which is countable, $f(\overline{F}_0)$ is G_δ in $f(F)$. Hence $f(\overline{F}_0 \cap \overline{F}_1) = f(\overline{F}_0) \cap f(\overline{F}_1)$ is G_δ in $f(F)$.

Now we define $Y^{(\alpha)}$ for each ordinal number α. Define $Y^{(0)} = Y$. Assume that $Y^{(\beta)}$ has been defined for $\beta < \alpha$. We define $Y^{(\alpha)}$: if α is a successor, say $\alpha = \beta + 1$, define

$$Y^{(\alpha)} = \overline{Y_0^{(\beta)}} \cap \overline{Y_1^{(\beta)}},$$

and if α is a limit, define $Y^{(\alpha)} = \cap_{\beta < \alpha} Y^{(\beta)}$.

Then $Y^{(\alpha)}$ is closed in Y and $\alpha < \beta$ implies $Y^{(\alpha)} \supset Y^{(\beta)}$. Therefore, since Y is hereditarily Lindelöf, it is easy to see that for some $\alpha^* < \omega_1, Y^{(\alpha^*)} = Y^{(\omega_1)}$.

We show $Y^{(\alpha^*)} = \emptyset$. Suppose $Y^{(\alpha^*)} \neq \emptyset$. Since $\alpha^* < \omega_1$, by Claim $f(Y^{(\alpha^*)})$ is G_δ in T. Hence $f(Y^{(\alpha^*)})$ is Čech-complete.

Since $Y^{(\alpha^*)}$ is closed and it belongs to $\prod$, by Lemma 2.7 we must have

$$Y^{(\alpha^*)} \neq \overline{Y_0^{(\alpha^*)}} \cap \overline{Y_1^{(\alpha^*)}} = Y^{(\alpha^*+1)}.$$

which is a contradiction. Thus, $Y^{(\alpha^*)} = \emptyset$. Therefore $Y = \bigcup_{\alpha<\alpha^*}(Y^{(\alpha)} - Y^{(\alpha+1)})$. Since $Y^{(\alpha)} - \overline{Y_1^{(\alpha)}} \subset Y_0^{(\alpha)}$ and $Y_0^{(\alpha)}$ is homeomorphic to $f(Y_0^{(\alpha)}), Y^{(\alpha)} - \overline{Y_1^{(\alpha)}}$ is a countable union of closed metrizable subspaces of $Y^{(\alpha)}$. So the same holds for $Y^{(\alpha)} - Y^{(\alpha+1)} = (Y^{(\alpha)} - \overline{Y_0^{(\alpha)}}) \cup (Y^{(\alpha)} - \overline{Y_1^{(\alpha)}})$ since $Y^{(\alpha)} - \overline{Y_0^{(\alpha)}} \subset Y_1^{(\alpha)}$. Therefore, since each $Y^{(\alpha)}$ is closed, Y is also a countable union of closed metrizable subspaces. Let $Y = \bigcup_{n\in N} E_n$, where E_n is closed metrizble. Then $E_n \in \prod$, and hence E_n is σ-locally compact. Thus, Y is σ-locally compact. This completes the proof of Theorem 2.8.

There have been defined several notions of generalized metric spaces (see, Gruenhage ([6]). The author does not know whether Theorem 2.8 is true for the classes of paracompact σ-spaces, stratifiable spaces or M_1-spaces. If a space Y is perfectly normal paracompact and admits T and $f : Y \to T$ described in Lemma 2.1, then the same proof shows that Theorem 2.8 holds for Y. But easy examples show that Lemma 2.1 does not hold even for M_1-spaces.

References

[1] A. V. Arhangel'skiĭ, *The closed image of a metric space can be condensed to a metric space*, Soviet Math. Dokl. **7** (1966), 1109-1112.

[2] M. Atsuji, *Normality of product spaces I*, in: Topics in General Topology, ed. by K. Morita and J. Nagata, North Holland (1989), 81-119.

[3] A. Bešlagić and K. Chiba, *Normality of products spaces*, Fund. Math. **128** (1987), 99-112.

[4] K. Chiba, *On products of normal spaces*, Rep. Fac. Sci. Shizuoka Univ. **9** (1974), 1-11.

[5] R. Engelking, *General Topology*, Berlin, 1989.

[6] G. Gruenhage, *Generalized metric spaces*, in: Handbook of Set-Theoretic Topology ed. by K. Kunen and J. E. Vaughan, North Holland (1984), 425-501.

[7] T. Hoshina, *Products of normal spaces with Lasňev spaces*, Fund. Math. **124** (1984), 143-153.

[8] ——, *Normality of product spaces II*, in: Topics in General Topology ed. by K. Morita and J. Nagata, North Holland (1989), 121-160.

[9] T. Ishii, *On product spaces and product mappings*, J. Math. Soc. Japan **18** (1966), 166-181.

[10] Y. Katuta, *On the normality of the product of a normal space with a paracompact space*, Gen. Top. Appl. **1** (1971), 295-319.

[11] N. Lašnev, *Continuous decompositions and closed mappings of metric spaces*, Soviet Math. Dokl. **6** (1965), 1504-1506.

[12] ——, *Closed images of metric spaces*, Soviet Math. Dokl. **7** (1966), 1219-1221.

[13] E. Michael, *The product of a normal space and a metric space need not be normal*, Bull. Amer. Math. Soc. **69** (1963), 375-376.

[14] K. Morita, *Products of normal spaces with metric spaces II*, Sci. Rep. Tokyo Kyoiku Daigaku **8** (1963), 87-92.

[15] ——, *On the product of paracompact spaces*, Proc. Japan Acad. **39** (1963),559-563.

[16] T. C. Przymusiński, *Products of normal spaces*, in: Handbook of Set-Theoretic Topology, ed. by K. Kunen and J.E. Vaughan, North Holland (1984), 781-826.

[17] M. E. Rudin and M. Starbird, *Products with a metric factor*, Gen. Top. Appl. **5** (1975), 235-348.

[18] R. Telgárski, *C-scattered and paracompact spaces*, Fund. Math. **73** (1971), 59-74.

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