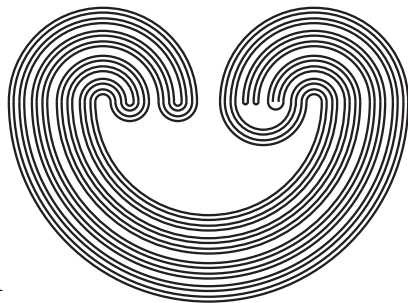


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## MULTI-VALUED FUNCTIONS AND TRIVIALITY

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**ABSTRACT.** We give in this paper a description of a new extension of the concept of contractible spaces to arbitrary topological spaces analogous to the notion of spaces of trivial shape. We shall use multi-valued functions with smaller and smaller images of points. Our notion is modeled on the property that every map from a contractible space is null-homotopic.

### INTRODUCTION

In [12] and in [3], it was shown recently that global (or shape) properties of spaces can be explored without the use of exterior objects (like neighborhoods in absolute neighborhood retracts or bonding spaces of polyhedral expansions) provided we use multi-valued functions instead of maps (i. e., continuous single-valued functions).

More precisely, the author has introduced in [3] a new category  $\mathcal{HM}$  whose objects are topological spaces and whose morphisms are homotopy classes of multi-nets. On compact metric spaces this category is isomorphic with Borsuk's shape category.

In this paper we shall explore an invariant of the category  $\mathcal{HM}$  which we name  $M\mathcal{D}$ -triviality, where  $\mathcal{D}$  is a class of spaces while the letter  $M$  suggests the use of multi-valued functions. This invariant corresponds to the notion of trivial shape and as we will see they are closely related. Both are generalizations of the notion of a contractible space from ordinary homotopy

theory.

In general, there are two different methods to examine the properties of a given space  $X$ . The first is to explore maps from some collection of spaces  $\mathcal{C}$  into  $X$  and the second is to study maps from  $X$  into members of  $\mathcal{C}$ . In the present paper we shall combine the two approaches in the study of the property "homotopic to a constant map".

In summary, using the concepts and methods of [3] (that have their origin in latest results of Sanjurjo [11] and [12]), we define the notion of an  $MD$ -trivial class of spaces and explore their properties that are similar to corresponding properties of fundamental absolute retracts [2] or spaces of trivial shape. In this way we have managed to improve many of known theorems about spaces of trivial shape.

### SMALL MULTI-VALUED FUNCTIONS

Let  $X$  and  $Y$  be topological spaces. By a *multi-valued function*  $F : X \rightarrow Y$  we mean a rule which associates a non-empty subset  $F(x)$  of  $Y$  to every point  $x$  of  $X$ .

Let  $\hat{Y}$  denote the collection of all normal covers of a topological space  $Y$  (see [1]). With respect to the refinement relation  $>$  the set  $\hat{Y}$  is a directed set. Two normal covers  $\sigma$  and  $\tau$  of  $Y$  are equivalent provided  $\sigma > \tau$  and  $\tau > \sigma$ . In order to simplify our notation we denote a normal cover and its equivalence class by the same symbol. Consequently,  $\hat{Y}$  also stands for the associated quotient set.

Let  $\tilde{Y}$  denote the collection of all finite subsets  $c$  of  $\hat{Y}$  which have a unique (with respect to the refinement relation) maximal element  $\tilde{c} \in \hat{Y}$ . We consider  $\tilde{Y}$  ordered by the inclusion relation and regard  $\hat{Y}$  as a subset of single-element subsets of  $\tilde{Y}$ . Notice that  $\tilde{Y}$  is a cofinite directed set [9,p.11].

Let  $F : X \rightarrow Y$  be a multi-valued function and let  $\xi \in \hat{X}$  and  $\sigma \in \hat{Y}$ . We shall say that  $F$  is a  $(\xi, \sigma)$ -map provided for every  $A \in \xi$  there is a  $C_A \in \sigma$  with  $F(A) \subset C_A$ . On the other hand,  $F$  is  $\sigma$ -small provided there is a  $\xi \in \hat{X}$  such that  $F$  is a

$(\xi, \sigma)$ -map.

Let  $F, G : X \rightarrow Y$  be multi-valued functions and let  $\sigma \in \hat{Y}$ . We shall say that  $F$  and  $G$  are  $\sigma$ -close and we write  $F \stackrel{\sigma}{\approx} G$  provided for every  $x \in X$  there is a  $C_x \in \sigma$  with  $F(x) \cup G(x) \subset C_x$ .

Let  $F, G : X \rightarrow Y$  be multi-valued functions between topological spaces and let  $\sigma$  be a normal cover of the space  $Y$ . We shall say that  $F$  and  $G$  are  $\sigma$ -homotopic and write  $F \stackrel{\sigma}{\simeq} G$  provided there is a  $\sigma$ -small multi-valued function  $H$  from the product  $X \times I$  of  $X$  and the unit segment  $I = [0, 1]$  into  $Y$  such that  $F(x) \subset H(x, 0)$  and  $G(x) \subset H(x, 1)$  for every  $x \in X$ . We shall say that  $H$  is a  $\sigma$ -homotopy that joins  $F$  and  $G$  or that it realizes the relation (or homotopy)  $F \stackrel{\sigma}{\simeq} G$ .

#### TRIVIAL CLASSES OF SPACES

Let  $\mathcal{C}$  and  $\mathcal{D}$  be classes of spaces. We shall say that  $\mathcal{C}$  is  $MD$ -trivial or that  $\mathcal{D}$  is  $CM$ -trivial provided for every member  $Y$  of  $\mathcal{D}$  and every normal cover  $\sigma$  of  $Y$  there is a normal cover  $\tau$  of  $Y$  such that every  $\tau$ -small multi-valued function from a member  $X$  of  $\mathcal{C}$  into  $Y$  is  $\sigma$ -homotopic to a constant map.

Important special cases are the following. If  $\mathcal{D}$  is a class consisting only of the space  $Y$  and a class  $\mathcal{C}$  is  $MD$ -trivial, then we say that the space  $Y$  is  $CM$ -trivial. This property was extensively studied in [4]. On the other hand, if  $\mathcal{C}$  is a class consisting only of the space  $X$  and a class  $\mathcal{D}$  is  $CM$ -trivial, then we say that the space  $X$  is  $MD$ -trivial. In the present paper we shall be mostly concerned with this notion. Finally, a space  $X$  is  $M$ -trivial provided for every normal cover  $\sigma$  of  $X$  the identity  $id_X$  is  $\sigma$ -homotopic to a constant map.

In the first two theorems we shall explore how our definitions depend on classes  $\mathcal{C}$  and  $\mathcal{D}$ .

Let  $\mathcal{B}$ ,  $\mathcal{C}$ , and  $\mathcal{D}$  be three classes of spaces. The pair  $(\mathcal{B}, \mathcal{C})$  is  $MD$ -surjective provided for every member  $Y$  of  $\mathcal{D}$  and every normal cover  $\sigma$  of  $Y$  there is a normal cover  $\tau$  of  $Y$  such that for every  $X \in \mathcal{B}$  and every  $\tau$ -small multi-valued function

$F : X \rightarrow Y$  there is a  $Z \in \mathcal{C}$ , a map  $h : X \rightarrow Z$ , and a  $\sigma$ -small multi-valued function  $G : Z \rightarrow Y$  with  $F \stackrel{\sigma}{\simeq} G \circ h$ .

For a normal cover  $\sigma$  of space  $X$ , let  $st(\sigma)$  denote the star of  $\sigma$ , let  $\sigma^*$  be the family of all  $\tau \in \hat{X}$  such that  $st(\tau)$  refines  $\sigma$ , and for a natural number  $n$ , let  $\sigma^{*n}$  denotes the set of all normal covers  $\tau$  of  $Z$  such that the  $n$ -th star  $st^n(\tau)$  of  $\tau$  refines  $\sigma$ .

**Theorem 1.** *Let  $\mathcal{B}$ ,  $\mathcal{C}$ , and  $\mathcal{D}$  be classes of spaces. If  $(\mathcal{B}, \mathcal{C})$  is  $MD$ -surjective and  $\mathcal{C}$  is  $MD$ -trivial, then  $\mathcal{B}$  is also  $MD$ -trivial.*

*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. Let  $\varrho \in \sigma^*$ . Since  $\mathcal{C}$  is  $MD$ -trivial, there is a refinement  $\pi \in \hat{Y}$  of  $\varrho$  such that every  $\pi$ -small multi-valued function from a member of  $\mathcal{C}$  into  $Y$  is  $\varrho$ -homotopic to a constant function. On the other hand, since  $(\mathcal{B}, \mathcal{C})$  is  $MD$ -surjective, there is a  $\tau \in \hat{Y}$  such that for every  $X \in \mathcal{B}$  and every  $\tau$ -small multi-valued function  $F : X \rightarrow Y$  there is a  $Z \in \mathcal{C}$  a map  $h : X \rightarrow Z$ , and a  $\pi$ -small multi-valued function  $G : Z \rightarrow Y$  with  $F \stackrel{\pi}{\simeq} G \circ h$ .

Consider a  $\tau$ -small multi-valued function  $F$  from a member  $X$  of  $\mathcal{B}$  into  $Y$ . Choose a space  $Z \in \mathcal{E}$ , a map  $h$ , and a multi-valued function  $G$  as above. Our choices imply that the function  $G$  is  $\varrho$ -homotopic to a constant function. Hence,  $F$  is  $\sigma$ -homotopic to a constant function.

Let  $\mathcal{C}$ ,  $\mathcal{D}$ , and  $\mathcal{E}$  be three classes of spaces. The pair  $(\mathcal{D}, \mathcal{E})$  is  $CM$ -injective provided for every member  $Y$  of  $\mathcal{D}$  and every normal cover  $\sigma$  of  $Y$  there is a  $Z \in \mathcal{E}$ , an  $\alpha \in \hat{Z}$  and a map  $h : Y \rightarrow Z$  such that for every  $X \in \mathcal{C}$  and every two multi-valued functions  $F, G : X \rightarrow Y$  the relation  $h \circ F \stackrel{\alpha}{\simeq} h \circ G$  implies  $F \stackrel{\sigma}{\simeq} G$ .

**Theorem 2.** *Let  $\mathcal{C}$  and  $\mathcal{D}$  be classes of spaces and let  $\mathcal{E}$  be a class of connected spaces. If  $(\mathcal{D}, \mathcal{E})$  is  $CM$ -injective and  $\mathcal{C}$  is  $M\mathcal{E}$ -trivial, then  $\mathcal{C}$  is also  $MD$ -trivial.*

*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. Since the pair  $(\mathcal{D}, \mathcal{E})$  is  $CM$ -injective, there is a  $Z \in \mathcal{E}$ ,

an  $\alpha \in \hat{Z}$ , and a map  $h : Y \rightarrow Z$  such that for every two multi-valued functions  $F, G : X \rightarrow Y$  the relation  $h \circ F \stackrel{\alpha}{\simeq} h \circ G$  implies  $F \stackrel{\alpha}{\simeq} G$ . Let  $\beta \in \alpha^*$ . Since  $\mathcal{C}$  is  $M\mathcal{E}$ -trivial, there is a  $\delta \in \hat{Z}$  such that every  $\delta$ -small multi-valued function from a member  $X$  of  $\mathcal{C}$  into  $Z$  is  $\beta$ -homotopic to a constant function. Let  $\tau = h^{-1}(\delta)$ .

Consider a space  $X$  from  $\mathcal{C}$  and a  $\tau$ -small multi-valued function  $F : X \rightarrow Y$ . The composition  $h \circ F$  is a  $\delta$ -small multi-valued function from  $X$  into  $Z$ . Since  $Z$  is connected, we can assume that there is a constant function  $G : X \rightarrow Y$  such that  $h \circ F \stackrel{\alpha}{\simeq} h \circ G$ . Our choices imply that  $F$  and  $G$  are  $\sigma$ -homotopic.  $\square$

In the next result we shall explore triviality of a space with respect to a class  $\mathcal{C}$  when we take the class  $\mathcal{C}$  to be either the class  $\mathcal{S}$  of all topological spaces or the class  $\mathcal{ANR}$  of all absolute neighborhood retracts. We shall also need the definition of O-spaces.

Recall [3] that a space  $X$  is an O-space provided it has an ANR-resolution  $p = \{p^a\} : X \rightarrow \{X_a, p_b^a, A\}$  [9], where each projection  $p^a$  is an onto map.

At present we do not know what is the real extend of O-spaces. From results in [10] it follows that inverse limits of inverse systems of compact Hausdorff spaces with onto bonding maps are O-spaces. In particular, all compact metric spaces are O-spaces. One can easily check that the examples of non-degenerate regular spaces with the property that every real-valued map on them is constant [6, p. 160] provide examples of spaces that are not O-spaces.

**Theorem 3.** *Every  $M$ -trivial topological space  $X$  is  $MS$ -trivial, and every  $MS$ -trivial space  $X$  is  $M\mathcal{ANR}$ -trivial. If  $X$  is an O-space, then all these forms of triviality are equivalent.*

*Proof:* We shall show that (1) an  $M$ -trivial space  $X$  is  $MS$ -trivial and that (2) an  $M\mathcal{ANR}$ -trivial O-space  $X$  is  $M$ -trivial. The other implications are obvious.

(1) Let a space  $Z$  and a normal cover  $\sigma$  of  $Z$  be given. Put  $\tau = \sigma$ . Consider a  $\tau$ -small multi-valued function  $F : X \rightarrow Z$ . Choose a normal cover  $\pi$  of  $X$  such that  $F$  is a  $(\pi, \tau)$ -map. Let  $H : X \times I \rightarrow X$  be a  $\pi$ -homotopy joining  $id_X$  with a constant function. Then  $F \circ H$  is a  $\sigma$ -homotopy joining  $F$  with a constant function.

(2) Let  $\sigma \in \hat{X}$ . Let  $\mathbf{p} = \{p^a\} : X \rightarrow \{X_a, p_b^a, A\}$  be an ANR-resolution of  $X$  with all projections  $p^a$  onto. By the property (B2) for  $\mathbf{p}$  (see [5, p.76]), there is an index  $a \in A$  and a normal cover  $\pi$  of  $X_a$  such that  $(p^a)^{-1}$  is a  $(\pi, \sigma)$ -map. By assumption, there is a normal cover  $\varrho$  of  $X_a$  such that every  $\varrho$ -small function from  $X$  into  $X_a$  is  $\pi$ -homotopic to a constant function. Since  $p^a : X \rightarrow X_a$  is surely  $\varrho$ -small, there is a  $\pi$ -homotopy  $H : X \times I \rightarrow X_a$  joining  $p^a$  with a constant function. The composition  $(p^a)^{-1} \circ H$  is a  $\sigma$ -homotopy joining  $id_X$  with a constant function.  $\square$

Observe that every contractible space  $X$  is  $MS$ -trivial. Indeed, let  $K : X \times I \rightarrow X$  be a homotopy joining the identity  $id_X$  with a constant function. Let  $Z$  be an arbitrary topological space, let  $\sigma$  be a normal cover of  $Z$  and let  $F$  be a  $\sigma$ -small multi-valued function from  $X$  into  $Z$ . The composition  $F \circ K$  is a  $\sigma$ -homotopy joining  $F$  with a constant function.

The converse will be true provided we assume that the space  $X$  has the following two properties:

- (A) For every normal cover  $\sigma$  of  $X$  there is a normal cover  $\tau$  of  $X$  such that every  $\tau$ -small multi-valued function into  $X$  is  $\sigma$ -close to a continuous single-valued function.
- (B) There is a normal cover  $\sigma$  of  $X$  with the property that  $\sigma$ -close maps into  $X$  are homotopic.

It was proved in [3] that every approximate polyhedron satisfies (A).

**Theorem 4** *If an  $MC$ -trivial space  $X$  belongs to the class  $\mathcal{C}$  and satisfies (A) and (B) above, then  $X$  is contractible.*

*Proof:* Let a normal cover  $\sigma$  of  $X$  satisfy the property from (B). Pick a normal cover  $\tau$  with respect to  $\sigma$  using (A). Since  $X$  is  $MC$ -trivial, there is a normal cover  $\rho$  of  $X$  such that every  $\rho$ -small multi-valued function from  $X$  into  $X$  is  $\tau$ -homotopic to a constant function. The fact that the identity  $id_X$  is clearly  $\rho$ -small imply that there is a  $\tau$ -small multi-valued function  $H : X \times I \rightarrow X$  joining  $id_X$  with a constant function  $k$ . Let  $K : X \times I \rightarrow X$  be a map  $\sigma$ -close to  $H$ . Then  $id_X \simeq H_0$  and  $H_1 \simeq k$  so that  $id_X \simeq k$ .  $\square$

### TRIVIALITY AND DOMINATION

We shall now show that  $MD$ -triviality is preserved by domination in the category  $\mathcal{HM}$ . In particular, it is an  $\mathcal{HM}$ -invariant. First, we must recall some definitions from [3] where the description of the category  $\mathcal{HM}$  is given.

Let  $X$  and  $Y$  be topological spaces. By a *multi-net* from  $X$  into  $Y$  we shall mean a collection  $\varphi = \{F_c \mid c \in \tilde{Y}\}$  of multi-valued functions  $F_c : X \rightarrow Y$  such that for every  $\gamma \in \hat{Y}$  there is a  $c \in \tilde{Y}$  with  $F_d \stackrel{\gamma}{\simeq} F_c$  for every  $d > c$ . We use functional notation  $\varphi : X \rightarrow Y$  to indicate that  $\varphi$  is a multi-net from  $X$  into  $Y$ . Let  $MN(X, Y)$  denote all multi-nets  $\varphi : X \rightarrow Y$ .

Two multi-nets  $\varphi = \{F_c\}$  and  $\psi = \{G_c\}$  between topological spaces  $X$  and  $Y$  are *homotopic* provided for every  $\gamma \in \hat{Y}$  there is a  $c \in \tilde{Y}$  such that  $F_d \stackrel{\gamma}{\simeq} G_d$  for every  $d > c$ .

The relation of homotopy is an equivalence relation on the set  $MN(X, Y)$ . The homotopy class of a multi-net  $\varphi$  is denoted by  $[\varphi]$  and the set of all homotopy classes by  $\mathcal{HM}(X, Y)$ .

In order to define the composition of homotopy classes of multi-nets we need the following careful selection of eight increasing functions associated with a multi-net.

Let  $\varphi = \{F_c\} : X \rightarrow Y$  be a multi-net. For every  $c \in \tilde{Y}$  there is an  $\bar{f}(c) \in \tilde{Y}$  such that for all  $d, e > \bar{f}(c)$  there is a normal cover  $\bar{f}(c, d, e)$  of  $X \times I$  and an  $(\bar{f}(c, d, e), \hat{c})$ -map joining  $F_d$  and  $F_e$ .

Let  $\mathcal{C} = \{(c, d, e) \mid c \in \tilde{Y}, d, e > \bar{f}(c)\}$ . Then  $\mathcal{C}$  is a subset



of  $\tilde{Y} \times \tilde{Y} \times \tilde{Y}$  that becomes a cofinite directed set when we define that  $(c, d, e) > (c', d', e')$  iff  $c > c', d > d',$  and  $e > e'$ .

Now, let  $f : \tilde{Y} \rightarrow \tilde{Y}$  be an increasing function such that  $f(c) > \tilde{f}(c),$   $c$  for every  $c \in \tilde{Y}$ . We shall use the same notation  $f$  for an increasing function  $f : \mathcal{C} \rightarrow \widehat{X \times I}$  such that  $f(c, d, e) > \tilde{f}(c, d, e)$  for every  $(c, d, e) \in \mathcal{C}$ . Let  $(c, d, e) \in \mathcal{C}$ . For the normal cover  $f(c, d, e)$  of  $X \times I$ , by [5, p. 358], there is a normal cover  $\varepsilon = \hat{f}(c, d, e)$  of  $X$  and a function  $r = \hat{f}(c, d, e) : \varepsilon \rightarrow \{2, 3, 4, \dots\}$  such that every set  $E \times [(i-1) \setminus rE, (i+1) \setminus rE]$ , where  $E \in \varepsilon$  and  $i = 1, 2, \dots, rE - 1$ , is contained in a member of  $f(c, d, e)$ .

Let  $\tilde{f} : \mathcal{C} \rightarrow \hat{X}$  be an increasing function with  $\tilde{f}(c, d, e) > \hat{f}(c, d, e)$  for every  $(c, d, e) \in \mathcal{C}$ . We shall use the shorter notation  $\tilde{f}(c)$  and  $\tilde{f}(c)$  for the covers  $\tilde{f}(c, f(c), f(c))$  and  $f(c, f(c), f(c))$ .

In [3] it was proved that there is an increasing function  $f^* : \tilde{Y} \rightarrow \hat{X}$  such that

- (1)  $f^*(c) > \tilde{f}(c)$  for every  $c \in Y$ , and
- (2)  $f^*$  is cofinal in  $\tilde{f}$ , i.e., for every  $(c, d, e) \in \mathcal{C}$  there is an  $m \in \tilde{Y}$  with  $f^*(m) > \tilde{f}(c, d, e)$ .

The above discussion shows that every multi-net  $\varphi : X \rightarrow Y$  determines eight functions denoted by  $\tilde{f}, f, \hat{f}, \tilde{f},$  and  $f^*$ . With the help of those functions we shall define the composition of homotopy classes of multi-nets as follows.

Let  $\varphi = \{F_c\} : X \rightarrow Y$  and  $\psi = \{G_s\} : Y \rightarrow Z$  be multi-nets. Let  $\chi = \{H_s\}$ , where  $H_s = G_{g(s)} \circ F_{f(g^*(s))}$  for every  $s \in \tilde{Z}$ . It can be shown that the collection  $\chi$  is a multi-net from  $X$  into  $Z$ . We now define the composition of homotopy classes of multi-nets by the rule  $[\{G_s\}] \circ [\{F_c\}] = [\{G_{g(s)} \circ F_{f(g^*(s))}\}]$ . Topological spaces as objects with homotopy classes of multi-nets between them as morphisms with the above composition form a category  $\mathcal{HM}$ .

Let  $\mathcal{B}$  and  $\mathcal{C}$  be classes of topological spaces. We say that the first  $\mathcal{HM}$ -dominates the second provided for every  $X \in \mathcal{C}$  there is a  $Y \in \mathcal{B}$  and multi-nets  $\varphi : X \rightarrow Y$  and  $\psi : Y \rightarrow X$

with the composition  $\psi \circ \varphi$  homotopic to the identity multi-net  $\iota_X$  on  $X$ .

The next theorem explains in which way the definition of  $MD$ -trivial class of spaces depends on the class  $\mathcal{D}$ .

**Theorem 5.** *Let  $\mathcal{D}$  be a class of spaces. A class of spaces  $\mathcal{C}$  which is  $\mathcal{HM}$ -dominated by an  $MD$ -trivial class  $\mathcal{B}$  is itself  $MD$ -trivial.*

*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. Let  $\pi \in \sigma^*$ . Since  $\mathcal{B}$  is  $MD$ -trivial, there is a normal cover  $\tau$  of  $Y$  with the property that every  $\tau$ -small multi-valued function from a member  $Z$  of  $\mathcal{B}$  into  $Y$  is  $\pi$ -homotopic to a constant function.

Consider a  $\tau$ -small multi-valued function  $H : X \rightarrow Y$  from a member  $X$  of  $\mathcal{C}$  into  $Y$ . Choose an  $\eta \in \hat{H}$  such that  $H$  is an  $(\eta, \tau)$ -map. Let  $\beta \in \eta^*$ . Since  $\mathcal{C}$  is  $\mathcal{HM}$ -dominated by  $\mathcal{B}$ , there is a  $Z \in \mathcal{B}$  and multi-nets  $\varphi : X \rightarrow Z$  and  $\psi : Z \rightarrow X$  with the composition  $\psi \circ \varphi$  homotopic to the identity multi-net  $\iota_X$  on  $X$ . Select an index  $a$  in  $\tilde{X}$  such that  $a > \{\beta\}$  and

$$(1) \quad G_x \circ F_z \stackrel{\beta}{\simeq} id_X,$$

where  $x = g(a)$ ,  $y = g^*(a)$ , and  $z = f(y)$ . The composition  $H \circ G_x$  is a  $\tau$ -small multi-valued function from  $Y$  into  $Z$ . By assumption, there is a  $\pi$ -small homotopy  $M : Y \times I \rightarrow Z$  joining  $H \circ G_x$  with a constant function. Choose a normal cover  $\mu$  of  $Y$  and a stacked normal cover  $\nu$  of  $Y \times I$  over  $\mu$  such that  $M$  is a  $(\nu, \pi)$ -map. Pick an index  $b > z$  so that  $F_b$  is  $\nu$ -small. Then  $M \circ (F_x \times id_I)$  is a  $\pi$ -small homotopy joining  $H \circ G_x \circ F_b$  with a constant function. Again, from the homotopy (1) and the above observation that  $G_x \circ F_b$  and  $G_x \circ F_z$  are  $\beta$ -homotopic, we now get at last that  $H$  is  $\sigma$ -homotopic to a constant function.  $\square$

**Corollary 1.** *Let  $\mathcal{D}$  be a class of spaces. A space  $\mathcal{HM}$ -dominated by an  $MD$ -trivial space is  $MD$ -trivial.*

**Theorem 6.** *Let a class of spaces  $\mathcal{D}$  be  $\mathcal{HM}$ -dominated by another such class  $\mathcal{E}$ . Let  $\mathcal{C}$  be a class of spaces. If  $\mathcal{C}$  is  $M\mathcal{E}$ -trivial, then it is also  $MD$ -trivial.*

*Proof:* Let a member  $Z$  of the class  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Z$  be given. Let  $\varrho \in \sigma^*$ . Since  $\mathcal{D}$  is  $\mathcal{HM}$ -dominated by  $\mathcal{E}$ , there is a  $W \in \mathcal{E}$  and multi-nets  $\varphi : Z \rightarrow W$  and  $\psi : W \rightarrow Z$  with  $\iota_Z \simeq \psi \circ \varphi$ . Hence, there is an index  $a > \{\varrho\}$  in  $\tilde{Z}$  and a  $\varrho$ -homotopy  $P : Z \times I \rightarrow Z$  with  $d \in P(d, 0)$  and  $G_x \circ F_z(d) \subset P(d, 1)$  for every  $d \in Z$ , where  $x = g(a)$ ,  $y = g^*(a)$ , and  $z = f(y)$ . Let  $\xi = \tilde{g}(a)$ . Then  $G_x$  is a  $(\xi, \varrho)$ -map. By assumption, there is a normal cover  $\eta$  of  $W$  with the property that  $\eta$  refines the cover  $y$  and every  $\eta$ -small multi-valued function from a member  $X$  of  $\mathcal{E}$  into  $W$  is  $\xi$ -homotopic to a constant function. Let  $b = f(\{\eta\})$ . Then  $F_b$  is an  $\eta$ -small multi-valued function and there is a  $y$ -homotopy  $K_z^b : Z \times I \rightarrow W$  with  $F_z(d) \subset K_z^b(d, 0)$  and  $F_b(d) \subset K_z^b(d, 1)$  for every  $d \in Z$ . Choose a normal cover  $\tau$  of  $Z$  and a stacked normal cover  $\mu$  of  $Z \times I$  over  $\tau$  with the property that  $P$  is a  $(\mu, \varrho)$ -map,  $K_z^b$  is a  $(\mu, y)$ -map, and  $F_b$  is a  $(\tau, \eta)$ -map.

Consider a  $\tau$ -small multi-valued function  $H$  from a member  $X$  of  $\mathcal{C}$  into  $Z$ . The composition  $F_b \circ H$  is an  $\eta$ -small multi-valued function from  $X$  into  $W$ . Hence, there is a  $\xi$ -homotopy  $Q : X \times I \rightarrow W$  and a  $q \in W$  such that  $F_b \circ H(p) \subset Q(p, 0)$  and  $q \in Q(p, 1)$  for every  $p \in X$ . Let  $s \in G_x(q)$  and let  $k_s$  be the constant function of  $X$  into the point  $s$ . It follows that the compositions  $P \circ (H \times id_I)$ ,  $G_x \circ K_z^b \circ (H \times id_I)$ , and  $G_x \circ Q$  are three  $\varrho$ -homotopies joining  $H$  and  $G_x \circ F_z \circ H$ ,  $G_x \circ F_z \circ H$  and  $G_x \circ F_b \circ H$ , and  $G_x \circ F_b \circ H$  and the constant function  $k_s$ , respectively. Hence,  $H \stackrel{\sigma}{\simeq} k_s$ .  $\square$

**Corollary 2.** *Let a class of spaces  $\mathcal{D}$  be  $\mathcal{HM}$ -dominated by another such class  $\mathcal{E}$ . If a space is  $M\mathcal{E}$ -trivial, then it is also  $MD$ -trivial.*

## CONNECTION WITH SHAPE

The notion of  $MD$ -triviality is closely related to the following shape invariant property. A space  $X$  is  $SD$ -trivial if some ANR-expansion  $p = \{p^a\} : X \rightarrow \{X_a, p_b^a, A\}$  of  $X$  satisfies: For every  $a \in A$  and every map  $f$  from  $X_a$  into a member  $Z$  of  $\mathcal{D}$  there is a  $b > a$  such that the composition  $f \circ p_b^a$  is null-homotopic.

One can show that every ANR-expansion of an  $SD$ -trivial space has the above property, that  $SD$ -triviality is preserved under shape domination, and that a space has trivial shape iff it is  $SANR$ -trivial.

The next theorem explains the relationship between our two different methods of extending the notion of contractibility to arbitrary topological spaces.

**Theorem 7.** (1) *If  $\mathcal{D}$  is a class of ANRs, then every  $MD$ -trivial space is also  $SD$ -trivial.*

(2) *If  $\mathcal{D}$  is a class of approximate polyhedra, then every  $SD$ -trivial space is  $MD$ -trivial.*

*Proof:* (1) Let  $\mathcal{D}$  be a class of ANRs and let  $X$  be an  $MD$ -trivial space. Let  $p = \{p^a\} : X \rightarrow \{X_a, p_b^a, A\}$  be an ANR-resolution of  $X$ . Let an index  $a \in A$  and a map  $f$  of  $X_a$  into a member  $Z$  of  $\mathcal{D}$  be given. Since  $Z$  is an ANR, there are normal covers  $\mu$  and  $\sigma$  such that  $\mu$ -close maps into  $Z$  are homotopic and  $\sigma$ -small multi-valued functions into  $Z$  are  $\mu$ -close to continuous single-valued functions. Since  $X$  is  $MD$ -trivial, there is a normal cover  $\tau$  of  $Z$  with the property that every  $\tau$ -small multi-valued function from  $X$  into  $Z$  is  $\sigma$ -homotopic to a constant function.

By assumption, there is a  $\sigma$ -homotopy  $H : X \times I \rightarrow Z$  and a point  $z_0 \in Z$  with  $f \circ p^a(x) \in H(x, 0)$  and  $z_0 \in H(x, 1)$  for every  $x \in X$ . Choose a continuous single-valued function  $h : X \times I \rightarrow Z$  with  $h \stackrel{\mu}{=} H$ . Define maps  $h_0, h_1 : Z \rightarrow X_a$  by  $h_0(z) = h(z, 0)$  and  $h_1(z) = h(z, 1)$  for every  $z \in Z$ . Then we have homotopies  $f \circ p^a \simeq h_0$ ,  $h_0 \simeq h_1$ , and  $h_1 \simeq k \circ p^a$ , where  $k$  is the constant function of  $X_a$  into the point  $z_0$ . Hence,  $f \circ p^a \simeq p^a$ . By the property (E2) of  $p$  (see [5, p. 48]), there is

an index  $b > a$  such that  $f \circ p_b^a \simeq k \circ p_b^a$ . It follows that  $f \circ p_b^a$  is null-homotopic.  $\square$

(2). Let a member  $Z$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Z$  be given. Let  $\xi \in \sigma^*$ . Since  $Z$  is an approximate polyhedron, there is a normal cover  $\tau$  of  $Z$  such that  $\tau$ -small multi-valued functions into  $Z$  are  $\xi$ -close to continuous single-valued functions.

Consider a  $\tau$ -small multi-valued function  $F$  of  $X$  into  $Z$ . Choose a continuous single-valued function  $f : X \rightarrow Z$  with  $f \stackrel{\xi}{\approx} F$ . Let  $\mathbf{p} = \{p^a\} : X \rightarrow \{X_a, p_b^a, A\}$  be an ANR-resolution of  $X$ . By the property (R1) for  $\mathbf{p}$  (see [9]), there is an index  $a \in A$  and a map  $f_a : X_a \rightarrow Z$  with  $f \stackrel{\xi}{\approx} f_a \circ p^a$ . By assumption, there is an index  $b > a$  and a homotopy  $h : X_b \times I \rightarrow Z$  joining  $f_a \circ p^a$  and a constant map. Define a multi-valued function  $H : X \times I \rightarrow Z$  by

$$H(x, t) = \begin{cases} \{h(p^b(x), t)\}, & x \in X, \quad t \neq 0 \\ F(x) \cup \{f_a \circ p^a(x)\}, & x \in X, \quad t = 0. \end{cases}$$

Then  $H$  is a  $\sigma$ -homotopy joining  $F$  with a constant function.  $\square$

As a consequence of the previous two theorems we get the following amusing characterization of spaces of trivial shape in the realm of O-spaces.

**Corollary 3.** (1) *An M-trivial space has trivial shape.*  
 (2) *An O-space  $X$  has trivial shape iff it is M-trivial.*

### TRIVIALITY AND $\mathcal{B}$ -LIKE SPACES

For the next result we must recall the notion of being like a class of spaces. Let  $\mathcal{B}$  be a class of topological spaces. A space  $X$  is *like a class  $\mathcal{B}$*  (or  *$\mathcal{B}$ -like*) provided for every normal cover  $\sigma$  of  $X$  there is a member  $Y$  of  $\mathcal{B}$  and a map  $g$  of  $X$  onto  $Y$  such that  $g^{-1} : Y \rightarrow X$  is a  $\sigma$ -small multi-valued function.

**Theorem 8.** *Let  $\mathcal{D}$  be a class of spaces. A class of spaces  $\mathcal{C}$  is  $MD$ -trivial if and only if  $\mathcal{C}$  is a class of  $\mathcal{B}$ -like spaces, where  $\mathcal{B}$  is an  $MD$ -trivial class of spaces.*

*Proof:* The implication  $(\implies)$  is obvious. In order to prove the opposite implication, let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. Since  $\mathcal{B}$  is  $MD$ -trivial, there is a normal cover  $\tau$  of  $Y$  such that every  $\tau$ -small multi-valued function  $F$  from a member  $W$  of  $\mathcal{B}$  into  $Y$  is  $\sigma$ -homotopic to a constant function.

Consider a  $\tau$ -small multi-valued function  $F$  from a member  $X$  of  $\mathcal{C}$  into  $Y$ . Pick a normal cover  $\xi$  of  $X$  so that  $F$  is a  $(\xi, \tau)$ -map. Since  $X$  is  $\mathcal{B}$ -like, there is a member  $W$  of  $\mathcal{B}$  and a map  $g$  of  $X$  onto  $W$  such that  $g^{-1}$  is  $\xi$ -small. The composition  $F \circ g^{-1}$  is a  $\tau$ -small multi-valued function of  $X$  into  $Y$ . It follows that there is a  $\sigma$ -homotopy  $H : W \times I \rightarrow Y$  and a  $y \in Y$  such that  $F \circ g^{-1}(w) \subset H(w, 0)$  and  $y \in H(w, 1)$  for every  $w \in W$ . Hence,  $H \circ (g \times id_I)$  is a  $\sigma$ -homotopy joining  $F$  and a constant function.  $\square$

**Corollary 4.** *Let  $\mathcal{D}$  be a class of spaces. A space is  $MD$ -trivial if and only if it is like an  $MD$ -trivial class of spaces.*

**Theorem 9.** *Let  $\mathcal{E}$  be a class of spaces. If a class of spaces  $\mathcal{C}$  is  $M\mathcal{E}$ -trivial and  $\mathcal{D}$  is a class of  $\mathcal{E}$ -like spaces, then  $\mathcal{C}$  is also  $MD$ -trivial.*

*Proof:* Let a member  $W$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $W$  be given. Since  $W$  is  $\mathcal{E}$ -like, there is a  $Z \in \mathcal{E}$  and a map  $g$  of  $W$  onto  $Z$  such that the inverse  $g^{-1} : Z \rightarrow W$  is a  $\sigma$ -small multi-valued function. Pick a normal cover  $\alpha$  of  $Z$  such that  $g^{-1}$  is an  $(\alpha, \sigma)$ -map. Since  $\mathcal{C}$  is  $M\mathcal{E}$ -trivial, there is a  $\beta \in \hat{Z}$  such that every  $\beta$ -small multi-valued function from a member of  $\mathcal{C}$  into  $Z$  is  $\alpha$ -homotopic to a constant function. Let  $\tau = g^{-1}(\beta)$ .

Consider a  $\tau$ -small multi-valued function  $F$  from a member  $X$  of  $\mathcal{C}$  into  $W$ . The composition  $goF$  is a  $\beta$ -small multi-valued function from a member of  $\mathcal{C}$  into  $Z$ . Let  $H : X \times I \rightarrow Z$  be

an  $\alpha$ -homotopy joining  $g \circ F$  with a constant function. Then  $g^{-1} \circ H$  is a  $\sigma$ -homotopy between  $F$  and a constant function.  $\square$

For a class of spaces  $\mathcal{D}$  let  $(\mathcal{D})$  denote the class of all  $\mathcal{D}$ -like spaces.

**Corollary 5.** *Let  $\mathcal{C}$  and  $\mathcal{D}$  be classes of spaces. If  $\mathcal{C}$  is  $MD$ -trivial, then  $\mathcal{C}$  is also  $M(\mathcal{D})$ -trivial.*

We let  $\mathcal{P}_f$  and  $\mathcal{KM}$  denote all finite polyhedra and all compact metric spaces, respectively. It is well-known that  $\mathcal{KM}$  is a class of  $\mathcal{P}_f$ -like spaces. Hence, from Corollary 5, we get the following.

**Corollary 6.** *If a class  $\mathcal{C}$  is  $M\mathcal{P}_f$ -trivial, then it is also  $M\mathcal{KM}$ -trivial.*

#### TRIVIALITY AND RIGHT PLACID MAPS

In the next two sections we shall look for classes of maps that will preserve the  $MD$ -trivial classes of spaces.

Let  $f : X \rightarrow Y$  be a map and let  $\tau$  be a normal cover of  $Y$ . We shall say that  $f$  is *right  $\tau$ -placid* provided for every  $\sigma \in X$  there is a  $\sigma$ -small multi-valued function  $G : Y \rightarrow X$  with  $f \circ G \stackrel{\tau}{\simeq} id_Y$ . A map  $f : X \rightarrow Y$  which is right  $\tau$ -placid for every  $\tau \in Y$  is called *right placid*. Observe that every map  $f : X \rightarrow Y$  which has a right homotopy inverse (i. e., for which there is a map  $g : Y \rightarrow X$  with  $f \circ g \simeq id_Y$ ) is right placid. The same is true if the map  $f$  has a right  $\mathcal{HM}$ -inverse.

Let  $\mathcal{D}$  and  $\mathcal{E}$  be classes of spaces. We shall say that  $\mathcal{D}$  is *rp-dominated* by  $\mathcal{E}$  provided for every  $Y \in \mathcal{D}$  and every  $\tau \in \hat{Y}$  there is an  $X \in \mathcal{E}$  and a right  $\tau$ -placid map  $f : X \rightarrow Y$ .

**Theorem 10.** *Let  $\mathcal{D}$  and  $\mathcal{E}$  be classes of spaces. If  $\mathcal{D}$  is rp-dominated by  $\mathcal{E}$  and a class of spaces  $\mathcal{C}$  is  $M\mathcal{E}$ -trivial, then  $\mathcal{C}$  is also  $MD$ -trivial.*

*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. Let  $\pi \in \sigma^*$ . Since  $\mathcal{D}$  is rp-dominated by  $\mathcal{E}$ , there is an  $X \in \mathcal{E}$  and a right  $\pi$ -placid map  $f : X \rightarrow Y$ . Let  $\alpha = f^{-1}(\pi)$ .

By assumption about  $\mathcal{C}$ , there is a normal cover  $\beta$  of  $X$  such that every  $\beta$ -small multi-valued function from a member of  $\mathcal{C}$  into  $X$  is  $\alpha$ -homotopic to a constant function. Choose a  $\beta$ -small multi-valued function  $G : Y \rightarrow X$  and an  $\alpha$ -homotopy  $H : Y \times I \rightarrow Y$  joining  $f \circ G$  and  $id_Y$ . Pick a  $\tau \in \hat{Y}$  and a stacked normal cover  $\xi$  of  $Y \times I$  over  $\tau$  such that  $G$  is a  $(\tau, \beta)$ -map and  $H$  is a  $(\xi, \alpha)$ -map.

Consider a  $\tau$ -small multi-valued function  $f$  from a member  $Z$  of  $\mathcal{C}$  into  $Y$ . The composition  $G \circ F$  is  $\beta$ -small so that there is a constant function  $k : Z \rightarrow X$  such that  $G \circ F \stackrel{\alpha}{\simeq} k$ . It follows that  $f \circ G \circ F \stackrel{\tau}{\simeq} f \circ k$ . But, the  $\pi$ -homotopy  $H \circ (F \times id_I)$  joins  $f \circ G \circ F$  with  $F$ . Hence,  $F$  is  $\sigma$ -homotopic to the constant function  $f \circ k$ .  $\square$

**Corollary 7.** *Let  $\mathcal{D}$  be a class of spaces. If  $f : X \rightarrow Y$  is a right placid map and  $X$  is  $MD$ -trivial, then  $Y$  is also  $MD$ -trivial.*

An important example of right placid maps provide refinable maps. We call an onto map  $f : X \rightarrow Y$  between topological spaces *refinable* provided for every normal cover  $\tau$  of  $Y$  and every normal cover  $\sigma$  of  $X$  there is an onto map  $g : X \rightarrow Y$  such that  $f$  and  $g$  are  $\tau$ -close and  $g^{-1}$  is a  $\sigma$ -small multi-valued function. We shall name the map  $g$  a  $(\sigma, \tau)$ -refinement of the map  $f$ . The notion of a refinable map between compact metric spaces was first defined by Jo Ford and James Rogers Jr.. The above extension to arbitrary topological spaces is particularly suitable for our theory. One can easily prove that refinable maps are right placid (see [4]).

The existence of a refinable map from a space  $X$  onto a space  $Y$  clearly implies that  $X$  is  $Y$ -like. Hence, as a consequence of Corollaries 4 and 7 we obtain the following improvement of cases (3) and (5) of Theorem (1.8) in [8].

**Corollary 8.** *Let  $\mathcal{D}$  be a class of spaces. Let  $f : X \rightarrow Y$  be a refinable map. Then  $X$  is  $MD$ -trivial if and only if  $Y$  is  $MD$ -trivial.*



## TRIVIALITY AND LEFT PLACID MAPS

Let  $f : X \rightarrow Y$  be a map and let  $\sigma$  be a normal cover of  $X$ . We shall say that  $f$  is *left  $\sigma$ -placid* provided there is a  $\sigma$ -small multi-valued function  $G : Y \rightarrow X$  with  $G \circ f \stackrel{\sigma}{\simeq} id_X$ . A map  $f : X \rightarrow Y$  which is left  $\tau$ -placid for every  $\tau \in \hat{Y}$  is called *left placid*. Observe that every map  $f : X \rightarrow Y$  which has a left homotopy inverse (i. e., for which there is a map  $g : Y \rightarrow X$  with  $g \circ f \simeq id_X$ ) is left placid. The same is true if the map  $f$  has a left  $\mathcal{HM}$ -inverse.

Let  $\mathcal{B}$  and  $\mathcal{C}$  be classes of spaces. We shall say that  $\mathcal{C}$  is *lp-dominated* by  $\mathcal{B}$  provided for every  $X \in \mathcal{C}$  and every  $\sigma \in X$  there is a  $Y \in \mathcal{B}$  and a left  $\sigma$ -placid map  $f : X \rightarrow Y$ .

**Theorem 11.** *Let  $\mathcal{B}$ ,  $\mathcal{C}$ , and  $\mathcal{D}$  be classes of spaces. If  $\mathcal{C}$  is lp-dominated by  $\mathcal{B}$  and the class  $\mathcal{B}$  is  $\mathcal{MD}$ -trivial, then  $\mathcal{C}$  is also  $\mathcal{MD}$ -trivial.*

*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. Let  $\pi \in \sigma^*$ . Since  $\mathcal{B}$  is  $\mathcal{MD}$ -trivial, there is a normal cover  $\tau$  of  $Y$  such that  $\tau$  refines  $\pi$  and every  $\tau$ -small multi-valued function from a member of  $\mathcal{B}$  into  $Y$  is  $\pi$ -homotopic to a constant function.

Consider a  $\tau$ -small multi-valued function  $F$  from a member  $X$  of  $\mathcal{C}$  into  $Y$ . Choose an  $\alpha \in X$  such that  $F$  is an  $(\alpha, \tau)$ -map. Since  $\mathcal{C}$  is lp-dominated by  $\mathcal{B}$ , there is a  $Z \in \mathcal{B}$  and a left  $\alpha$ -placid map  $f : X \rightarrow Z$ . Let  $G : Z \rightarrow X$  be an  $\alpha$ -small multi-valued function with  $G \circ f \stackrel{\alpha}{\simeq} id_X$ . Let  $H : X \times I \rightarrow X$  be an  $\alpha$ -homotopy joining  $G \circ f$  and  $id_X$ . Observe that the composition  $F \circ G$  is a  $\tau$ -small multi-valued function from  $Z$  into  $Y$ . It follows that there is a constant function  $k : X \rightarrow Y$  such that  $F \circ G \stackrel{\pi}{\simeq} k$ . But, the composition  $F \circ H$  is a  $\tau$ -homotopy joining  $F \circ G \circ f$  and  $F$ . Hence,  $F$  is  $\sigma$ -homotopic to the constant function  $k$ .  $\square$

**Corollary 9.** *Let  $\mathcal{D}$  be a class of spaces. If  $f : X \rightarrow Y$  is a left placid map and the space  $Y$  is  $\mathcal{MD}$ -trivial, then  $X$  is also  $\mathcal{MD}$ -trivial.*

The  $MD$ -trivial spaces are also inversely preserved under left  $\mathcal{D}$ -placid maps. Here we say that a map  $f : X \rightarrow Y$  between topological spaces is *left  $\mathcal{D}$ -placid* provided for every member  $Z$  of  $\mathcal{D}$  and every normal cover  $\sigma$  of  $Z$  there is a normal cover  $\tau$  of  $Z$  such that for every  $\tau$ -small multi-valued function  $G : X \rightarrow Z$  there is a  $\sigma$ -small multi-valued function  $H : Y \rightarrow Z$  with  $G \stackrel{e}{\simeq} H \circ f$ .

**Theorem 12.** *Let  $\mathcal{D}$  be a class of spaces. If  $f : X \rightarrow Y$  is a left  $\mathcal{D}$ -placid map and  $Y$  is  $MD$ -trivial, then  $X$  is also  $MD$ -trivial.*

*Proof:* Let a member  $Z$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Z$  be given. Let  $\pi \in \sigma^*$ . Since  $Y$  is  $MD$ -trivial, there is a normal cover  $\varrho$  of  $Z$  such that  $\varrho$  refines  $\pi$  and every  $\varrho$ -small multi-valued function from  $Y$  into  $Z$  is  $\pi$ -homotopic to a constant function. Finally, since  $f$  is left  $\mathcal{D}$ -placid and  $Z \in \mathcal{D}$ , there is a normal cover  $\tau$  of  $Z$  with the property that for every  $\tau$ -small multi-valued function  $F : X \rightarrow Z$  there is a  $\varrho$ -small multi-valued function  $G : Y \rightarrow Z$  with  $F \stackrel{e}{\simeq} G \circ f$ .

Consider a  $\tau$ -small multi-valued function  $F$  from  $X$  into  $Z$ . Choose a  $\varrho$ -small multi-valued function  $G$  as above. Our choices imply that  $G$  is  $\pi$ -homotopic to a constant function. It follows that  $G \circ f$  is  $\pi$ -homotopic to a constant function. Hence,  $F$  is  $\sigma$ -homotopic to a constant function.  $\square$

Another important example of left placid maps provide inclusions  $i_A, X$  of the  $M$ -retracts  $A$  of a space  $X$ . Here, we say that a subset  $A$  of a space  $X$  is an  *$M$ -retract* of  $X$  provided for every normal cover  $\sigma$  of  $A$  there is a  $\sigma$ -small multi-valued function  $R : X \rightarrow A$  such that  $a \in R(a)$  for every  $a \in A$ . Hence, the following is a consequence of Corollary 9.

**Corollary 10.** *Let  $\mathcal{D}$  be a class of spaces. An  $M$ -retract of an  $MD$ -trivial space is itself  $MD$ -trivial.*

## TRIVIALITY AND EQUIVALENCES

For the class of  $MD(\ell, m, n)$ -equivalences that we define next, another result similar to Corollary 8 can be proved. Let  $\ell, m, n$  be either zero or natural numbers with  $m, n \geq \ell$ . A map  $f : X \rightarrow Y$  between topological spaces is an  $MD(\ell, m, n)$ -equivalence provided for every member  $Z$  of  $\mathcal{D}$ , every normal cover  $\sigma$  of  $Z$ , and every  $\sigma$ -small multi-valued function  $G : X \rightarrow Z$  there is a  $st^\ell(\sigma)$ -small multi-valued function  $H : Y \rightarrow Z$ , unique up to a  $st^n(\sigma)$ -homotopy, such that  $G \stackrel{st^m(\sigma)}{\simeq} H \circ f$ . We use the name  $M(\ell, m, n)$ -equivalence if  $\mathcal{D}$  is the class  $\mathcal{S}$  of all topological spaces.

**Theorem 13.** *Let  $\ell, m$ , and  $n$  be either zero or natural numbers with  $m, n \geq \ell$ . Let  $f : X \rightarrow Y$  be an  $MD(\ell, m, n)$ -equivalence. Then  $X$  is  $MD$ -trivial if and only if  $Y$  is  $MD$ -trivial.*

*Proof:* ( $\Rightarrow$ ) Let a member  $Z$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Z$  be given. Choose a normal cover  $\pi$  of  $Z$  such that  $st^n(\pi)$  refines  $\sigma$ . Since  $X$  is  $MD$ -trivial, there is a  $\tau \in \hat{Z}$  such that  $\tau$  refines  $\pi$  and every  $\tau$ -small multi-valued function from  $X$  into  $Z$  is  $st^m(\pi)$ -homotopic to a constant function.

Consider a  $\tau$ -small multi-valued function  $H$  from  $Y$  into  $Z$ . The composition  $H \circ f : X \rightarrow Z$  is  $\tau$ -small so that it is  $st^m(\pi)$ -homotopic to a constant function  $K$  of  $X$  into the point  $z_0 \in Z$ . Let  $J$  denote the constant function of  $Y$  into  $z_0$ . Then  $H$  and  $J$  are two  $st^\ell(\pi)$ -small multi-valued function such that the compositions  $H \circ f$  and  $J \circ f$  are both  $st^n(\pi)$ -homotopic to  $H \circ f$ . It follows that  $H$  and  $J$  are  $st^n(\pi)$ -homotopic. Hence,  $H$  is  $\sigma$ -homotopic to a constant function.  $\square$

( $\Leftarrow$ ) This follows easily from Theorem 12 because every  $MD(\ell, m, n)$ -equivalence is left  $\mathcal{D}$ -placid.  $\square$

**Corollary 11.** *Let  $\ell, m$ , and  $n$  be either zero or natural numbers with  $m, n \geq \ell$ . Let  $f : X \rightarrow Y$  be an  $M(\ell, m, n)$ -equivalence. Then  $X$  is  $M$ -trivial if and only if  $Y$  is  $M$ -trivial.*

## TRIVIALITY AND TAMENESS

In the presence of tameness type properties we can conclude that triviality with respect to one class of spaces implies triviality with respect to another class.

Let  $\mathcal{B}$ ,  $\mathcal{C}$ , and  $\mathcal{D}$  be classes of spaces. The class  $\mathcal{D}$  is called *BCM-tame* provided for every  $Z \in \mathcal{D}$  and every normal cover  $\sigma$  of  $Z$  there is a normal cover  $\tau$  of  $Z$  with the property that for every  $X \in \mathcal{B}$  and every  $\tau$ -small multi-valued function  $F : X \rightarrow Z$  we can find a  $Y \in \mathcal{C}$ , a map  $f : X \rightarrow Y$ , and a  $\sigma$ -small multi-valued function  $G : Y \rightarrow Z$  with  $F \stackrel{\sigma}{\simeq} G \circ f$ .

**Theorem 14.** *Let  $\mathcal{B}$ ,  $\mathcal{C}$ , and  $\mathcal{D}$  be classes of spaces. If  $\mathcal{D}$  is BCM-tame and  $\mathcal{C}$  is MD-trivial, then  $\mathcal{B}$  is also MD-trivial.*

*Proof:* Let a member  $Z$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Z$  be given. Let  $\varrho \in \sigma^*$ . Since  $\mathcal{C}$  is MD-trivial, there is a normal cover  $\pi$  of  $Z$  such that  $\pi$  refines  $\varrho$  and every  $\pi$ -small multi-valued function from a member of  $\mathcal{C}$  into  $Z$  is  $\varrho$ -homotopic to a constant function. On the other hand, since  $\mathcal{D}$  is BCM-tame, there is a normal cover  $\tau$  of  $Z$  with the property that for every  $\tau$ -small multi-valued function  $F$  of a member  $X$  of  $\mathcal{B}$  we can find a  $Y \in \mathcal{C}$ , a map  $f : X \rightarrow Y$ , and a  $\pi$ -small multi-valued function  $G : Y \rightarrow Z$  with  $F \stackrel{\pi}{\simeq} G \circ f$ .

Consider a  $\tau$ -small multi-valued function  $F$  of a member  $X$  of  $\mathcal{B}$  into  $Z$ . Select a  $Y \in \mathcal{C}$ , a map  $f$ , and a function  $G$  as above. Let  $H : X \times I \rightarrow Z$  be a  $\pi$ -homotopy joining  $F$  with  $G \circ f$ . By assumption, there is a  $\varrho$ -homotopy  $K : Y \times I \rightarrow Z$  that joins  $G$  with a constant function. Hence, the  $\varrho$ -homotopies  $H$  and  $K \circ (f \times id_I)$  together allow to conclude that  $F$  is  $\sigma$ -homotopic to a constant function.  $\square$

The following special case of Theorem 14 is worthy of mention. In order to state it, we shall first introduce the notion of a  $\Phi M$ -extensor.

Let  $\Phi$  be a class of maps. A class of spaces  $\mathcal{D}$  is a  $\Phi M$ -extensor provided for every  $Z \in \mathcal{D}$  and every  $\sigma \in \hat{Z}$  there is a  $\tau \in \hat{Z}$  with the property that for every map  $f : X \rightarrow Y$  in  $\Phi$

and every  $\tau$ -small multi-valued function  $F : X \rightarrow Z$  there is a  $\sigma$ -small multi-valued function  $G : Y \rightarrow Z$  with  $F \stackrel{\sigma}{\simeq} G \circ f$ .

For a class of maps  $\Phi$ , let  $\Phi'$  and  $\Phi''$  denote classes of domains and codomains of members of  $\Phi$ , respectively.

One can easily check that a class of spaces  $\mathcal{D}$  is  $\Phi'\Phi''M$ -tame when  $\mathcal{D}$  is a  $\Phi M$ -extensor. Hence, we obtain the following corollary.

**Corollary 12.** *Let  $\Phi$  be a class of maps. If a class of spaces  $\mathcal{D}$  is both a  $\Phi M$ -extensor and  $\Phi''M$ -trivial, then it is also  $\Phi'M$ -trivial.*

Let  $\mathcal{C}$ ,  $\mathcal{D}$  and  $\mathcal{E}$  be classes of topological spaces. The class  $\mathcal{C}$  is called  $M\mathcal{D}\mathcal{E}$ -tame provided for every member  $Y$  of  $\mathcal{D}$  and every normal cover  $\sigma$  of  $Y$  there is a normal cover  $\tau$  of  $Y$ , a  $Z \in \mathcal{E}$ , and a map  $f : Z \rightarrow Y$  such that for every  $\tau$ -small multi-valued function  $F : X \rightarrow Y$  from a member  $X$  of  $\mathcal{C}$  into  $Y$  and every normal cover  $\kappa$  of  $Z$  we can find a  $\kappa$ -small multi-valued function  $G : X \rightarrow Z$  with  $F \stackrel{\sigma}{\simeq} f \circ G$ .

**Theorem 15.** *Let  $\mathcal{C}$ ,  $\mathcal{D}$ , and  $\mathcal{E}$  be classes of spaces. If  $\mathcal{C}$  is both  $M\mathcal{D}\mathcal{E}$ -tame and  $M\mathcal{E}$ -trivial, then it is also  $M\mathcal{D}$ -trivial.*

*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. Let  $\varrho \in \sigma^*$ . Since  $\mathcal{C}$  is  $M\mathcal{D}\mathcal{E}$ -tame, there is a normal cover  $\tau$  of  $Y$ , a  $Z \in \mathcal{E}$ , and a map  $f : Z \rightarrow Y$  such that  $\tau$  refines  $\varrho$  and for every  $\tau$ -small multi-valued function  $F : X \rightarrow Y$  from a member  $X$  of  $\mathcal{C}$  into  $Y$  and every normal cover  $\kappa$  of  $Z$  we can find a  $\kappa$ -small multi-valued function  $G : X \rightarrow Z$  with  $F \stackrel{\sigma}{\simeq} f \circ G$ .

Consider a  $\tau$ -small multi-valued function  $F$  from a member  $X$  of  $\mathcal{C}$  into  $Y$ . Let  $\alpha = f^{-1}(\tau)$ . Since  $\mathcal{C}$  is also  $M\mathcal{E}$ -trivial and  $Z$  is a member of  $\mathcal{E}$ , there is a normal cover  $\kappa$  of  $Z$  with the property that every  $\kappa$ -small multi-valued function from  $X$  into  $Z$  is  $\alpha$ -homotopic to a constant function. Choose a  $\kappa$ -small multi-valued function  $G : X \rightarrow Z$ , a  $\varrho$ -homotopy  $H : X \times I \rightarrow Z$  joining  $F$  with  $f \circ G$ , and an  $\alpha$ -homotopy  $K : X \times I \rightarrow Z$  joining  $G$  with a constant function  $k$ . Using the  $\varrho$ -homotopies

$H$  and  $f \circ K$  we conclude that  $F$  is  $\sigma$ -homotopic to the constant function  $f \circ k$ .  $\square$

The following special case of Theorem 15 is worthy of mention. In order to state it, we shall first introduce the notion of a  $CM$ -bundle.

Let  $\mathcal{C}$  be a class of spaces. A class of maps  $\Phi$  is a  $CM$ -bundle provided for every  $f : X \rightarrow Y$  map in  $\Phi$  and every  $\sigma \in \hat{Y}$  there is a  $\tau \in \hat{Y}$  with the property that for every  $\tau$ -small multi-valued function  $F : Z \rightarrow Y$  from a member  $Z$  of  $\mathcal{C}$  into  $Y$  and every  $\pi \in \hat{X}$  there is a  $\pi$ -small multi-valued function  $G : Z \rightarrow X$  with  $F \simeq f \circ G$ .

One can easily check that a class of spaces  $\mathcal{C}$  is  $M\Phi''\Phi'$ -tame when  $\Phi$  is a  $CM$ -bundle. Hence, we obtain the following corollary.

**Corollary 13.** *Let  $\Phi$  be a class of maps. If a class of spaces  $\mathcal{C}$  is  $\Phi'M$ -trivial and  $\Phi$  is a  $CM$ -bundle, then  $\mathcal{C}$  is  $\Phi''M$ -trivial.*

### TRIVIALITY AND MOVABILITY

In the presence of the movability type condition we can obtain that triviality with respect to each part  $\mathcal{C}_a$  of the class  $\mathcal{C}$  implies triviality with respect to the entire class  $\mathcal{C}$ .

Let  $\mathcal{C}$  and  $\mathcal{D}$  be a classes of spaces. The class  $\mathcal{D}$  is  $CM$ -movable provided for every member  $Y$  of  $\mathcal{D}$  and every normal cover  $\sigma$  of  $Y$  there is a normal cover  $\tau$  of  $Y$  such that for every  $\tau$ -small multi-valued function from a member  $X$  of  $\mathcal{C}$  into  $Y$  and every normal cover  $\varrho$  of  $Y$  there is a  $\varrho$ -small multi-valued function  $G$  from  $X$  into  $Y$  which is  $\sigma$ -homotopic to  $F$ .

**Theorem 16.** *Let  $\{\mathcal{C}_a \mid a \in A\}$  be a collection of classes of spaces. Let  $\mathcal{C}$  denote the union  $\cup_{a \in A} \mathcal{C}_a$ . If a class of spaces  $\mathcal{D}$  is  $CM$ -movable and  $\mathcal{C}_a M$ -trivial for every  $a \in A$ , then  $\mathcal{D}$  is also  $CM$ -trivial.*

*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $X$  be given. Let  $\xi \in \sigma^*$ . Since  $\mathcal{D}$  is  $CM$ -movable, there is a normal

cover  $\tau$  of  $Y$  such that  $\tau$  refines  $\xi$  and for every  $\tau$ -small multi-valued function  $F$  from a member  $X$  of  $\mathcal{C}$  into  $Y$  and every normal cover  $\varrho$  of  $Y$  there is a  $\varrho$ -small multi-valued function  $G : X \rightarrow Y$  with  $F \stackrel{\xi}{\simeq} G$ .

Consider a member  $X$  of  $\mathcal{C}$  and a  $\tau$ -small multi-valued function  $F : X \rightarrow Y$ . Choose an  $a \in A$  so that  $X \in \mathcal{C}_a$ . Since  $\mathcal{D}$  is  $\mathcal{C}_a M$ -trivial, there is a normal cover  $\varrho$  of  $Y$  with the property that every  $\varrho$ -small multi-valued function from a member of  $\mathcal{C}_a$  into  $Y$  is  $\tau$ -homotopic to a constant function. Now, we select a  $\varrho$ -small multi-valued function  $G$  as above. Then  $G$  is  $\tau$ -homotopic to a constant function  $k$ . Hence,  $F \stackrel{\varrho}{\simeq} k$ .  $\square$

**Theorem 17.** *Let  $\{\mathcal{D}_a \mid a \in A\}$  be a collection of classes of spaces and let  $\mathcal{C}$  be a class of spaces. Let  $\mathcal{D}$  denote the union  $\bigcup_{a \in A} \mathcal{D}_a$ . If  $\mathcal{C}$  is  $MD$ -movable and  $MD_a$ -trivial for every  $a \in A$ , then  $\mathcal{C}$  is also  $MD$ -trivial.*

*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. Let  $\pi \in \sigma^*$ . Since  $\mathcal{C}$  is  $MD$ -movable, there is a normal cover  $\tau$  of  $Y$  such that  $\tau$  refines  $\pi$  and for every  $\tau$ -small multi-valued function  $F$  from a member  $X$  of  $\mathcal{C}$  into  $Y$  and every normal cover  $\varrho$  of  $Y$  there is a  $\varrho$ -small multi-valued function  $G : X \rightarrow Y$  with  $F \stackrel{\pi}{\simeq} G$ .

Consider a member  $X$  of  $\mathcal{C}$  and a  $\tau$ -small multi-valued function  $F : X \rightarrow Y$ . Choose an  $a \in A$  so that  $Y \in \mathcal{D}_a$ . Since  $X$  is  $MD_a$ -trivial, there is a normal cover  $\varrho$  of  $Y$  with the property that every  $\varrho$ -small multi-valued function  $G$  from  $X$  into  $Y$  is  $\tau$ -homotopic to a constant function. Now, we select a  $\varrho$ -small multi-valued function  $G$  as above. Then  $G$  is  $\tau$ -homotopic to a constant function  $k$ . Hence,  $F \stackrel{\varrho}{\simeq} k$ .  $\square$

## TRIVIALITY AND PRODUCTS

The notion of an  $MD$ -trivial class of spaces  $\mathcal{C}$  behaves well with respect to the coproducts and products. However, for products, it is necessary to make an extra assumption because

normal covers of the product are not always derived from products of normal covers of factors.

**Theorem 18.** *Let  $\{C_a \mid a \in A\}$  be a collection of non-empty classes of spaces. Let  $\mathcal{C}$  denote the class of all coproducts  $\coprod_{a \in A} X_a$  with  $X_a \in C_a$  for every  $a \in A$ . If a class of spaces  $\mathcal{D}$  is  $CM$ -trivial, then  $\mathcal{D}$  is also  $C_a M$ -trivial for every  $a \in A$ .*

*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. Since  $\mathcal{D}$  is  $CM$ -trivial, there is a normal cover  $\tau$  of  $Y$  such that every  $\tau$ -small multi-valued function from a member of  $\mathcal{C}$  into  $Y$  is  $\sigma$ -homotopic to a constant function.

Consider an  $a \in A$ , an  $X_a \in C_a$ , and a  $\tau$ -small multi-valued function  $F : X_a \rightarrow Y$ . For every  $b \in A \setminus \{a\}$ , select a space  $X_b \in C_b$ . Let  $X = \coprod_{c \in A} X_c$ . Observe that  $X$  is a member of  $\mathcal{C}$ . Let  $y_0 \in Y$ . Define  $G : X \rightarrow Y$  by

$$G(x) = \begin{cases} F(x), & \text{if } x \in X, \text{ and } x \in X_a, \\ \{y_0\}, & \text{if } x \in X, \text{ and } x \notin X_a. \end{cases}$$

Then  $G$  is a  $\tau$ -small multi-valued function from  $X$  into  $Y$ . Let  $H$  be a  $\sigma$ -homotopy joining  $G$  and a constant function. The restriction of  $H$  to the subspace  $X_a$  shows that  $F$  is  $\sigma$ -homotopic to a constant function.  $\square$

We shall say that a collection  $\{\mathcal{D}_a \mid a \in A\}$  of classes of spaces is *entwined* provided whenever  $X_a \in \mathcal{D}_a$  for  $a \in A$  then an open cover  $\sigma$  of the product  $X = \prod_{a \in A} X_a$  is normal if and only if there are finitely many indices  $a, \dots, n$  in  $A$  and normal covers  $\sigma_a, \dots, \sigma_n$  of  $X_a, \dots, X_n$ , respectively, such that the open cover

$$\{U \times \dots \times W \times \prod \{X_s \mid s \in A \setminus \{a, \dots, n\}\} \mid U \in \sigma_a, \dots, W \in \sigma_n\}$$

of  $X$  refines  $\sigma$ .

**Theorem 19.** *Let  $\mathcal{C}$  be a class of spaces. If  $\{\mathcal{D}_a \mid a \in A\}$  is an entwined collection of  $CM$ -trivial classes of spaces  $\mathcal{D}_a$ , then the class  $\mathcal{D}$  of all products  $Y = \prod_{a \in A} Y_a$ , where  $Y_a \in \mathcal{D}_a$  for every  $a \in A$ , is also  $CM$ -trivial.*



*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. By assumption, there are finitely many indices  $a, \dots, n$  in  $A$  and normal covers  $\sigma_a, \dots, \sigma_n$  of  $Y_a, \dots, Y_n$ , respectively, such that the open cover

$$\eta = \{U \times \dots \times W \times \prod \{Y_s \mid s \in A \setminus \{a, \dots, n\}\} \mid \\ U \in \sigma_a, \dots, W \in \sigma_n\}$$

of  $Y$  refines  $\sigma$ . Since each space  $Y_a, \dots, Y_n$  is  $\mathcal{CM}$ -trivial, there are normal covers  $\tau_a, \dots, \tau_n$  of  $Y_a, \dots, Y_n$ , respectively, with the property that every  $\tau_s$ -small multi-valued function from a member of  $\mathcal{C}$  into  $Y_s$  is  $\sigma_s$ -homotopic to a constant function ( $s = a, \dots, n$ ). Let

$$\tau = \{U \times \dots \times W \times \prod \{Y_s \mid s \in A \setminus \{a, \dots, n\}\} \mid \\ U \in \tau_a, \dots, W \in \tau_n\}.$$

Then  $\tau$  is a normal cover of  $Y$ .

Consider a  $\tau$ -small multi-valued function  $F$  from a member  $X$  of  $\mathcal{C}$  into  $Y$ . For each  $s = a, \dots, n$ , the composition  $\pi_s \circ F$  of  $F$  with the projection  $\pi_s$  of  $Y$  onto the factor  $Y_s$  is a  $\tau_s$ -small multi-valued function of  $X$  into  $Y_s$ . Let  $H^s : X \times I \rightarrow Y_s$  be a  $\sigma_s$ -homotopy joining  $\pi_s \circ F$  with a constant function  $k_s : X \rightarrow Y_s$ . Define a multi-valued function  $H : X \times I \rightarrow Y$  by

$$H(x, t) = H^a(x, t) \times \dots \times H^n(x, t) \times \prod \{Y_s \mid s \in A \setminus \{a, \dots, n\}\}$$

for every  $(x, t) \in X \times I$ . Then  $H$  is a  $\sigma$ -homotopy joining  $F$  with a constant function.  $\square$

### WEAK TRIVIALITY

The notion of  $\mathcal{MD}$ -triviality for a class  $\mathcal{C}$  has also the following weaker form that could be interesting for some type of situations. Many of the previous results with only minor modifications can be proved for the new concept.

Let  $\mathcal{C}$  and  $\mathcal{D}$  be classes of spaces. We shall say that  $\mathcal{C}$  is *weakly MD-trivial* provided for every map  $f : X \rightarrow Y$  from a member  $X$  of  $\mathcal{C}$  into a member  $Y$  of  $\mathcal{D}$  and every normal cover  $\sigma$  of  $Y$  there is a point  $y_0 \in Y$  and a  $\sigma$ -small multi-valued

function  $H : X \times I \rightarrow Y$  such that  $H(x, 0) = \{f(x)\}$  and  $H(x, 1) = \{y_0\}$  for every  $x \in X$ .

We shall first consider the question when does this weaker form agrees with the original. It turns out that the following class of spaces provides an easy answer to this problem.

Let  $\mathcal{C}$  and  $\mathcal{D}$  be a classes of spaces. The class  $\mathcal{C}$  is *internally MD-movable* provided for every  $Y \in \mathcal{D}$  and every normal cover  $\sigma$  of  $Y$  there is a normal cover  $\tau$  of  $Y$  such that for every  $\tau$ -small multi-valued function  $F$  from a member  $X$  of  $\mathcal{C}$  into  $Y$  there is a map  $g$  from  $X$  into  $Y$  which is  $\sigma$ -homotopic to  $F$ .

**Theorem 20.** *Let  $\mathcal{C}$  and  $\mathcal{D}$  be classes of topological spaces. If the class  $\mathcal{C}$  is both internally MD-movable and weakly MD-trivial, then  $\mathcal{C}$  is MD-trivial.*

*Proof:* Let a member  $Y$  of  $\mathcal{D}$  and a normal cover  $\sigma$  of  $Y$  be given. Let  $\varrho \in \sigma^*$ . Since  $\mathcal{C}$  is internally MD-movable, there is a  $\tau \in \hat{Y}$  with the property that every  $\tau$ -small multi-valued function  $F$  from a member  $X$  of  $\mathcal{C}$  into  $Y$  is  $\varrho$ -homotopic to a continuous single-valued function  $g : X \rightarrow Y$ .

Consider a member  $X$  of  $\mathcal{C}$  and a  $\tau$ -small multi-valued function  $F : X \rightarrow Y$ . Choose a map  $g : X \rightarrow Y$  as above. Since  $\mathcal{C}$  is weakly MD-trivial,  $g$  is  $\varrho$ -homotopic to a constant function. Hence,  $F$  is  $\sigma$ -homotopic to a constant function.  $\square$

One version of Corollary 11 for weakly  $\mathcal{CM}$ -trivial spaces uses a notion of a  $\mathcal{C}_p$ -extensor, where  $\mathcal{C}_p$  is a class of pairs. Recall that a space  $X$  is a  $\mathcal{C}_p$ -extensor provided for every member  $(A, B)$  of  $\mathcal{C}_p$  and every map  $f : B \rightarrow X$  there is a map  $g : A \rightarrow X$  with  $f = g|_B$ . For example, Hu's notion of an absolute extensor for the (weakly hereditary topological) class  $\mathcal{C}$  is a special case of this definition (see [7, p.35]). Also, a compact space  $X$  is a  $\mathcal{B}_p$ -extensor, where  $\mathcal{B}_p$  denotes all pairs  $(\beta B, B)$  consisting of a completely regular space  $B$  and it's Stone-Ćech compactification  $\beta B$ . Hence, we obtain the following corollary. We use  $\mathcal{K}$  and  $\mathcal{CR}$  to denote classes of all compact and

all completely regular spaces, respectively. If we use the Wallman extensions, then we get an analogous statement involving all quasi-compact and all  $T_1$ -spaces.

**Corollary 14.** *If a class of compact spaces  $\mathcal{D}$  is weakly  $\mathcal{KM}$ -trivial, then  $\mathcal{D}$  is also weakly  $\mathcal{CRM}$ -trivial.*

*Added in proof:* In a recent paper entitled "Shape Theory Intrinsically" (to appear in *Publicacions Matemàtiques*) the author has shown that Morita's shape category and the category  $\mathcal{HM}$  are isomorphic. Using techniques from this paper it is possible to eliminate the assumption on  $O$ -spaces from all our results where they appear. In particular,  $M$ -trivial spaces coincide with spaces of trivial shape.

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