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# H-BOUNDED SETS

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ABSTRACT. H-bounded sets were introduced by Lambrinos in 1976. Recently they have proven to be useful in the study of extensions of Hausdorff spaces. In this context the question has arisen: Is every H-bounded set the subset of an H-set? This was originally asked by Lambrinos along with the questions: Is every H-bounded set the subset of a countably H-set? and Is every countably H-bounded set the subset of a countably H-set? In this paper, an example is presented which gives a no answer to each of these questions. Some new characterizations and properties of H-bounded sets are also examined.

## 1. INTRODUCTION

The concept of an H-bounded set was defined by Lambrinos in [1] as a weakening of the notion of a bounded set which he defined in [2]. H-bounded sets have arisen naturally in the author's study of extensions of Hausdorff spaces. In [3] it is shown that the notions of S- and $\theta$-equivalence defined by Porter and Votaw [6] on the upper-semilattice of H-closed extensions of a Hausdorff space are equivalent if and only if every closed nowhere dense set is an H-bounded set. As a consequence, it is shown that a Hausdorff space has a unique Hausdorff extension if and only if the space is non-H-closed, almost H-closed, and every closed nowhere dense set is H-bounded. It is shown in Proposition 3.2 below that for locally H-closed (and hence almost H-closed spaces) a set is H-bounded if and only if it is the subset of an H-closed set. In [1] Lambrinos asks if this is always the case. More specifically he asks:

**Question 1.1.** *Is every H-bounded set the subset of an H-set?*

In light of these applications, it is important to examine H-bounded sets more throughly. The paper by Lambrinos [1] examines many basic properties of H-bounded sets. Lambrinos also examines a number of related concepts, several of which involve countable covering properties. In conjunction with these he asks:

**Question 1.2.** *Is every H-bounded set the subset of a countably H-set?*

**Question 1.3.** *Is every countably-H-bounded set the subset of a countably H-set?*

In the second section of this paper, an example is presented which gives a negative answer to the three questions above. In the third section, H-bounded sets are examined more closely. In particular a new characterization of H-bounded sets is given and the structure of H-bounded sets is explored.

This introduction will conclude with definitions for the concepts discussed above and some basic facts. All spaces under consideration are Hausdorff spaces. In [2] Lambrinos defined a subset of a space to be *bounded* if every open cover of the space has a finite subcollection which covers the set. In [1] he weakened this notion is several ways. Two of these are H-bounded and countably H-bounded.

**Definition 1.4.** [1] *A subset $A$ of a space $X$ is said to be an* H-bounded set *if every open cover of $X$ by sets which are open in $X$ has a finite subcollection whose closures in $X$ cover $A$.*

**Definition 1.5.** [1] *A subset $A$ of a space $X$ is said to be a* countably H-bounded set *if every countable open cover of $X$*

*by sets which are open in $X$ has a finite subcollection whose closures in $X$ cover $A$.*

It should be noted that Lambrinos used the terminology "almost bounded" and "countably almost bounded" for "H-bounded" and "countably H-bounded". This paper will use "H" instead of "almost" to parallel the related terms of H-closed and H-set.

A space is H-closed if it is closed in every Hausdorff space in which it is embedded. (H-closed abbreviates Hausdorff closed.) The question of what an appropriate definition for a subset to be H-closed leads to two distinct concepts.

**Definition 1.6.** *A subset $A$ of a space $X$ is said to be an* H-closed subset *if every cover of $A$ by sets which are open in $A$ has a finite subcollection whose closures in $A$ cover the set $A$. Thus an H-closed subset is an H-closed subspace.*

**Definition 1.7.** [9] and [4] *A subset $A$ of a space $X$ is said to be an* H-set *if every cover of $A$ by sets which are open in $X$ has a finite subcollection whose closures in $X$ cover $A$.*

H-closed sets are internally defined, while H-sets are externally defined. H-bounded sets are externally defined but deal with coverings of the whole space.

Another definition due to Lambrinos is a countably H-set.

**Definition 1.8.** [9] and [1] *A subset $A$ of a space $X$ is said to be a* countably H-set *if every countable cover of $A$ by sets which are open in $X$ has a finite subcollection whose closures in $X$ cover $A$.*

Lambrinos [1] does an excellent job of examining fundamentals of H-bounded sets and of comparing H-bounded sets, H-sets, countably H-bounded sets, countably H-sets, and a number of other types of sets not discussed here. A few facts about these various types of sets will be mentioned for the reader's convenience and the reader is refered to [1] for a comprehensive treatment of this material.

It follows from the definitions that compact sets are H-closed, H-closed sets are H-sets, and H-sets are H-bounded. The next example shows that the implications do not reverse.

**Example 1.9.** *Let $X = \omega \cup \omega \times \omega \cup \{p\}$ with the following topology:*

(1) *Points in $\omega \times \omega$ are isolated.*
(2) *A basic neighborhood of $n \in \omega$ has the form $\{n\} \cup \{(n,m) : m \in \omega \setminus F\}$ for some $F \in [\omega]^{<\omega}$.*
(3) *A basic neighborhood of $p$ has the form $\{p\} \cup \{(n,m) : m \in \omega, n \in \omega \setminus F\}$ .*

It is easy to see that $X$ is H-closed but not compact, $\omega \cup \{p\}$ is an H-set which is not H-closed, and $\omega$ is a closed, H-bounded set which is not an H-set.

It also follows from the definitions that every subset of a compact set, an H-closed set, or an H-set is H-bounded. Question 1.1 is asking the converse. Since every subset of a compact set is H-bounded, H-bounded sets need not be closed (which is not the case for compact sets, H-closed sets, or H-sets). Our primary interest, however, will be with closed, H-bounded sets.

Later the following filter characterizations will be needed.

**Proposition 1.10** *Let $A \subseteq X$.*

(1) *$A$ is an H-closed set if and only if every open filter on $A$ is fixed.*
(2) *$A$ is an H-set if and only if every open filter on $X$ which meets $A$ has an adherent point in $A$.*
(3) *$A$ is an H-bounded set if and only if every open filter on $X$ which meets $A$ is fixed.*

## 2. Example

An example of a space which has a closed, H-bounded subset which is not contained in any H-set is presented in this section.

**Example 2.1.** *Let $X = \omega \cup (\omega \times \omega \times \omega^*) \cup \omega^*$ topologized as follows:*

(1) *Points in* $\omega \times \omega \times \omega^*$ *are isolated.*
(2) *A basic open neighborhood of* $n \in \omega$ *has the form:* $\{n\} \cup \bigcup\{\{n\} \times (\omega \setminus F_u) \times \{u\} : u \in \omega^*\}$ *where each* $F_u \in [\omega]^{<\omega}$.
(3) *A basic open neighborhood of* $u \in \omega^*$ *has the form:* $A \times \omega \times \{u\} \cup \{u\}$ *where* $A \in u$.

A good way to visualize this space is to consider the space in Example 1.9, but with the topology of the point at infinity changed to represent a free ultrafilter on $\omega$ instead of a cofinite filter. Our space then is $2^c$ copies of this space (each with its point at infinity corresponding to a different free ultrafilter) with their $\omega$ sets identified.

To simplify the discussion, each subset of the form $\omega\times\omega\times\{u\}$ will be called a *plank* and referred to as the $u^{th}$ plank. $u$ is the point at infinity for the $u^{th}$ plank. $\omega^*$ is the set of all points at infinity. Within a plank, a set of the form $\{n\} \times \omega \times \{u\}$ will be called a *column* and be referred to as the $n^{th}$ column of the $u^{th}$ plank. Thus a basic open neighborhood of $n$ consists of $n$ along with all but finitely many points of the $n^{th}$ column of each plank. In the context of the space $X$, the symbol $\omega$ will always refer to the copy of $\omega$ which is not part of any plank or $\omega^*$.

**Proposition 2.2.** *The set* $\omega$ *in* $X$ *is a closed, H-bounded, and countably H-bounded subset of* $X$.

*Proof:* It is easy to see that $\omega$ is closed, for every point in $X \setminus \omega$ has a basic open neighborhood that misses $\omega$.

We claim that $\omega$ is H-bounded and hence countably H-bounded. To see this begin by considering an arbitrary open cover of $\omega^*$ by basic open sets in $X$ (i.e., by sets of the form $A_u \times \omega \times \{u\} \cup \{u\}$ where $A_u \in u$). Within the space $\beta\omega$, $A_u \in u$ which implies that $u \in cl_{\beta\omega}(A_u)$ which is clopen. Then $\{cl_{\beta\omega}(A_u) : u \in \omega^*\} \cup \{\{n\} : n \in \omega\}$ is an open cover of the compact space $\beta\omega$. There exists $u_1, \ldots, u_k, n_1, \ldots, n_m$ such that

$$\beta\omega = \bigcup\{cl_{\beta\omega}(A_{u_i}) : i = 1,\ldots,k\} \cup \bigcup\{\{n_j\} : j = 1,\ldots,m\}.$$

Thus $\bigcup\{cl_{\beta\omega}(A_{u_i}) : i = 1,\ldots,k\}$ covers all but finitely many points of $\omega$. But $cl_{\beta\omega}(A_{u_i}) \cap \omega = A_{u_i}$, so $\bigcup\{A_{u_i} : i = 1,\ldots,k\}$ covers all but finitely many points of $\omega$. Returning again to the space $X$, notice that $\bigcup\{cl_X((A_{u_i} \times \omega \times \{u_i\}) \cup \{u_i\}) : i = 1,\ldots,k\}$ covers all but finitely many points of $\omega$. Thus every open cover of $X$ has a finite subcollection whose closures cover $\omega$. □

**Proposition 2.3.** *An infinite H-set of $X$ meets only finitely many columns of finitely many planks and meets both $\omega^*$ and $\omega$ in finite sets.*

*Proof.* Suppose $B$ is an infinite H-set. Consider covers consisting of basic open sets. First consider the intersection of $B$ with $\omega^*$. No point of $B \cap \omega^*$ is in the closure of a basic neighborhood of any point of $X \setminus \omega^*$. Also if $C$ is a basic open neighborhood of $u \in \omega^*$, then $u$ is the only element of $\omega^*$ in $cl(C)$ (because the neighborhoods of $u$ "live" only on the $u^{th}$ plank). Thus $B \cap \omega^*$ is finite.

Next consider the intersection of $B$ with a single plank, say the $u^{th}$ plank. The closures of basic open neighborhoods of points in other planks are just the points themselves. The closures of basic neighborhoods of points at infinity of other planks do not pick up any points in any other plank (only points in $\omega$). Thus only basic neighborhoods of points in $\omega$ and the point $u$ have closures which contribute to covering $B$. If $B$ meets an infinite number of columns, then there is a set $A_u \in u$ such that $A_u \times \omega \times \{u\} \cup \{u\}$ misses an infinite number of the columns that $B$ meets. Since $B$ is an H-set, only a finite number of neighborhoods of points in $\omega$ can help cover what the neighborhood of $u$ did not cover. But each basic neighborhood of a point in $\omega$ covers only part of a single column in a given plank. Thus $B$ can meet only finitely many columns of any given plank.

The set $B$ can meet only a finite number of planks. To see this, suppose $B$ meets an infinite number of planks. It was just shown that $B$ meets each plank in a finite number of columns and that one can always choose neighborhoods of the points in $\omega^*$ to miss these columns. Thus all but finitely many of these points must be picked up by neighborhoods of points in $\omega$. So not only does $B$ meet each plank in a finite number of columns, it meets each plank in the same finite number of columns. Suppose some column, say the $n^{th}$ column, is met by $B$ in an infinite number of planks, say $\{u(\alpha) : \alpha \in \kappa\}$ for some infinite cardinal $\kappa$. $F_{u(\alpha)}$ may be chosen so that $\{n\} \times \omega \setminus F_{u(\alpha)} \times \{u\}$ misses at least one point in $B$ in the $n^{th}$ column. Defining $F_u$ arbitrarily on the remaining planks gives an open neighborhood of $n$ that misses an infinite number of points of $B$, only finitely many of which can be picked up in some other way by the closures of a finite number of basic open sets. Therefore $B$ can meet each column on only a finite number of planks, and since $B$ meets only finitely many columns, it follows that $B$ meets only finitely many planks.

Finally if $B$ meets $\omega$ in an infinite set, then since $B \cap \omega^*$ is finite, choose basic open neighborhoods of the points in $B \cap \omega^*$ whose closures miss an infinite number of points in $B \cap \omega$. The closures of basic neighborhoods of points in the planks miss $\omega$ and the closures of basic neighborhoods of points in $\omega$ pick up only one point in $\omega$. Since $B$ is an H-set, it is possible to use only neighborhoods of a finite number of points of $\omega$, which then leaves infinitely many points of $B$ uncovered. Therefore $B$ must meet $\omega$ in a finite set. $\square$

**Proposition 2.4.** *An infinite countably H-set of $X$ meets only finitely many columns of finitely many planks and meets both $\omega^*$ and $\omega$ in finite sets.*

*Proof:* The proof of this proposition is similar to the proof of Proposition 2.3, but with the necessary modifications to work with countable covers. To simplify notation, basic open neighborhoods of points in $\omega^*$ will be denoted as $C_u(A) =$

$A \times \omega \times \{u\} \cup \{u\}$ where $u \in \omega^*$ and $A \in u$. Sometimes $C_u$ will be used to indicate any basic open neighborhood of $u$ if the set $A$ is not important. Also of interest will be neighborhoods of points in $\omega$ which consist of the point $n$ and all of the $n^{th}$ column. They will be denoted as $S_n = \{n\} \cup \{n\} \times \omega \times \omega^*$.

Let $B$ be an infinite countably H-set. Suppose $B \cap \omega^*$ is infinite. Let $b$ be a countably infinite subset of $B \cap \omega^*$. Cover $B$ by $\{S_n : n \in \omega\} \cup \{C_u(A_u) : u \in b\} \cup \bigcup\{C_u(A_u) : u \in \omega^* \setminus b\}$ where $A_u$ is any set in $u$. This is a countable open cover of $B$. Since the $S_n$ are clopen and if $v \in \omega^* \setminus \{u\}$ then $v \notin cl_X(C_u(A_u))$ and if $v \in b$ then $v \notin cl \bigcup\{C_u(A_u) : u \in \omega^* \setminus b\}$, it follows that there is no finite subcollection whose closures cover. Thus $B$ can contain only finitely many elements of $\omega^*$.

Next consider $B$'s intersection with the $u^{th}$ plank. Suppose that $B$ meets an infinite number of columns. Choose $A_u$ such that $C_u(A_u)$ misses infinitely many of the columns that $B$ touches. Arbitrarily pick $A_v$ for $v \in B \cap \omega^* \setminus \{u\}$ (which is finite). Cover $B$ with $\{C_v(A_v) : v \in B \cap \omega\} \cup \{S_n : n \in \omega\}$. This is a countable cover, but infinitely many $S_n$ are needed in any subcollection whose closures cover. Thus $B$ meets only finitely many columns of the $u^{th}$ plank.

Now suppose that $B$ meets infinitely many planks. If $B$ is covered with $\{C_u(A_u) : u \in B \cap \omega\} \cup \{S_n : n \in \omega\}$, then it is evident that in order to have a finite subcollection with closures dense, at most the same finitely many columns can meet $B$ in each plank. Next suppose that $B$ hits the $n^{th}$ column on infinitely many planks. Let $b$ be an countably infinite subset of $\omega^*$ such that if $u \in b$ then $B$ hits the $n^{th}$ column of the $u^{th}$ plank. Let $F_u = \omega$ if $B$ misses the $n^{th}$ column of the $u^{th}$ plank, and let $F_u = \omega \setminus \{x\}$ otherwise where $x$ is an element of $B$ in the $n^{th}$ column of the $u^{th}$ plank. Let $T_n = \{n\} \cup \bigcup\{\{m\} \times F_u \times \{u\} : u \in \omega^*\}$. Then the cover $\{C_u(A_u) : u \in \omega\} \cup \{F_m : m \in \omega \setminus \{u\}\} \cup \{T_n\} \cup \{\{n\} \times \{m\} \times \{u\} : m \in \omega,\ u \in b\}$ is countable and infinitely many of the singletons are required in order for the closures of any subcollection to cover. Thus $B$ meets only finitely many planks.

Finally suppose $B \cap \omega$ is infinite. Choose $A_u \subset \omega$ for $u \in B \cap \omega^*$ such that $A_u \in u$ for each $u$ and $\omega \cup \{A_u : u \in B \cap \omega^*\} \cup \{S_n : n \in \omega\}$ is a countable open cover of $B$ with no finite dense subcollection. Therefore $B$ meets at most a finite subset of $\omega$. $\square$

**Corollary 2.5.** *The set $\omega$ in $X$ is not a subset of an H-set nor is it a subset of any countably H-set of $X$.*

*Proof:* Every infinite H-set or countably H-set meets at most finitely many points of $\omega$. $\square$

## 3. Characterizations and Structure

In this section, a new characterization of H-bounded sets is presented and the structure of H-bounded sets is examined.

The H-closed property is defined in terms of embeddings. The next theorem shows that H-bounded sets also may be characterized in terms of embeddings. The author is not aware of a similar characterization for H-sets.

**Proposition 3.1.** *Let $A \subset X$.*

(1) *$A$ is H-closed if and only if $A$ is closed in every Hausdorff space in which it is embedded.*
(2) *$A$ is H-bounded if and only if $cl_X(A)$ is closed in every space in which $X$ is embedded. For closed, H-bounded sets this becomes: $A$ is a closed, H-bounded set if and only if $A$ is closed in every Hausdorff space in which $X$ is embedded.*

*Proof:* 1. This follows from the definition of H-closed and the fact that an H-closed subset is an H-closed space.
2. *Necessity.* Suppose $X$ is embedded in $Y$. If $p \in Y \setminus X$ let $O^p = \{U \cap X : U \in \tau(Y), \text{ and } p \in U\}$. $O^p$ is a free open filter on $X$. It follows from Proposition 1.10 that if $A$ is H-bounded then $O^p$ does not meet $A$. Hence, there is a $Y$-neighborhood of $p$ that misses $A$, and so $p \notin cl_Y(A)$. Since no points of $Y \setminus X$ are in $cl_Y(A)$, it follows that $cl_Y(A) = cl_X(A)$.
*Sufficiency.* Suppose, by way of contradiction, that $A \subseteq X$

is not H-bounded. Then there is a free open filter $\mathcal{F}$ on $X$ meeting $A$. Let $Y = X \cup \{p\}$ be topologized as follows: A set $U$ is open if $p \in U$ and $U \cap X \in \mathcal{F}$ or if $p \notin U$ then $U \in \tau(X)$. As $\mathcal{F}$ is free, this topology on $Y$ is Hausdorff. Clearly, $O^p = \mathcal{F}$ and $\mathcal{F}$ meets $A$ so every neighborhood of $p$ meets $A$. Therefore $p$ is in $cl_Y(A)$, but not in $cl_X(A)$, which contradicts the assumption. □

Due to the usefulness of H-bounded sets in the study of extensions of Hausdorff spaces, an important problem is to understand the structure of H-bounded sets. Section 2 showed that while all subsets of H-sets are H-bounded, not all H-bounded sets are the subsets of H-sets. The next result contrasts with the example of the previous section.

**Proposition 3.2.** *If $X$ is locally H-closed then every H-bounded set is contained in an H-closed set.*

*Proof:* Let $A$ be an H-bounded set; then $cl(A)$ is also H-bounded. Let $\mathcal{G} = \cap\{\mathcal{F} : \mathcal{F}$ is a free open filter$\}$. Porter [5] has shown that if $X$ is locally H-closed then 1) $\mathcal{G}$ is free and 2) $\mathcal{G}$ is the filter generated by $\{U \in \tau : X \setminus U$ is H-closed$\}$. Since $cl(A)$ is H-bounded, Proposition 1.10 implies that $\mathcal{G}$ does not meet $cl(A)$. There exists a $U \in \mathcal{G}$ such that $U \cap cl(A) = \emptyset$ and $X \setminus U$ is H-closed. This implies that $U \subseteq X \setminus cl(A)$, hence $cl(A) \subseteq X \setminus U$. Thus $A$ is contained in an H-closed set. □

The converse to the previous proposition is not true. For example, let $X$ be a Tychonoff space which is not locally compact. Then $A \subseteq X$ is H-bounded if and only if $cl_X A$ is compact.

Another type of structure problem is whether an H-bounded set can be decomposed into the union of sets with properties which are more thoroughly understood. Using the following result of Lambrinos, some progress in this direction can be made.

**Proposition 3.3.** [1] *Let $A$ be a regular closed, H-bounded subset of $X$. Then $A$ is an H-closed subset of $X$.*

**Corollary 3.4.** *Every closed H-bounded set has the form $B \cup H$ where $B$ is an H-closed, regular closed set and $H$ is a closed, nowhere dense, H-bounded set. This union need not be disjoint; however, every closed, H-bounded set has the form $B \cup K$ where $B \cap K = \emptyset$, $B$ is H-closed, regular closed, and $K$ is H-bounded and nowhere dense (but not necessarily closed).*

*Proof:* Let $A$ be closed and H-bounded. Let $B = cl(int(A))$. $B$ is regular closed and a subset of $A$ and by Proposition 3.3 is H-closed. Now let $H = A \setminus int(A)$ (or let $K = A \setminus cl(int(A))$). $H$ is closed nowhere dense and a subset of $A$, hence H-bounded ($K$ is nowhere dense and H-bounded). □

Thus the difference between an H-bounded set and an H-closed set is a nowhere dense set. It is desirable to have a description of what H-bounded sets look like. This corollary reduces the search to nowhere dense H-bounded sets. This problem is related to the open question of characterizing the closed subspaces of an H-closed space that are H-closed. It is interesting to compare this corollary to the following result by Woods:

**Proposition 3.5.** [7, 7.6(g)(4)] or [10] *A subspace $X$ of an H-closed space is H-closed if and only if it is of the form $B \cup H$, where $B$ is regular closed and $H$ is an H-closed nowhere dense subset of $X$.*

Under certain conditions an H-bounded set is an H-set. This is examined next. Recall the notion of $\theta$-closure. If $A \subset X$ then $cl_\theta A = \{x \in X : cl(U) \cap A \neq \emptyset$ for all neighborhoods $U$ of $x\}$. $A$ is said to be $\theta$-closed if $A = cl_\theta A$. From Proposition 1.10 it follows that a subset $A$ of a space is H-bounded if and only if every open filter meeting $A$ has an adherent point in $cl_\theta A$. The following is an immediate corollary:

**Corollary 3.6.** [1] *A $\theta$-closed, H-bounded set is an H-set.*

Recall that a space is Urysohn if distinct points have disjoint closed neighborhoods.

**Proposition 3.7.** [9] *An H-set in an Urysohn space is $\theta$-closed.*

The previous two fact imply the following characterization of H-sets in terms of H-bounded sets in Urysohn spaces:

**Corollary 3.8.** *In an Urysohn space, a subset is an H-set if and only if it is $\theta$-closed and H-bounded.*

A well known result (see [8] or [7,4S(6)]) is that if every closed subspace of a space is H-closed then the space is compact. There is similar result relating H-bounded subsets to H-closed spaces.

**Proposition 3.9.** *A space $X$ is H-closed if and only if every proper, closed subset of $X$ is H-bounded.*

*Proof:* Necessity follows from the fact that every subset of an H-closed set is H-bounded. Conversely let $\mathcal{U}$ be any open cover of nonempty sets of $X$. Pick $U \in \mathcal{U}$. $X \setminus U$ is a proper closed subset and by assumption is H-bounded. There exists a finite number of elements of $\mathcal{U}$ whose closures cover $X \setminus U$. These together with $U$ form a finite subcollection of $\mathcal{U}$ whose closures cover $X$. □

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