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PROBLEM SECTION

Contributed Problems

The Problems Editor invites anyone who has published a paper in *Topology Proceedings* or has attended a Spring or Summer Topology Conference to submit problems to this section. They need not be related to any articles which have appeared in *Topology Proceedings* or elsewhere, but if they are, please provide full references. Please define any terms not in a general topology text nor in referenced articles.

Problems which are stated in, or relevant to, a paper in this volume are accompanied by the title of the paper where further information about the problem may be found. Comments of the proposer or submitter of the questions are so noted; comments of the Problems Editor are not spcially noted. Information on the status of previously posed questions is always welcome. Submission of questions and comments by email in TEX form is strongly encouraged, either to topolog@mail.auburn.edu or directly to the Problems Editor at mayer@math.uab.edu.

B. Generalized Metric Spaces and Metrization

41. (H. Bennett and D. Lutzer, *Off-diagonal metrization theorems*) Is it consistently true that if X is a Lindelöf LOTS that is paracompact off of the diagonal, then X has a σ-point finite base?

42. (Bennett and Lutzer, *ibid.*) Can there be a Souslin space (*i.e.*, a non-separable LOTS with countable cellularity, no completeness or connectedness assumed) such that $X^2 - \Delta$ is paracompact? Hereditarily paracompact?

43. (Bennett and Lutzer, *ibid.*) Suppose X is a LOTS that is first countable and hereditarily paracompact off of the diagonal (*i.e.*, $X^2 - \Delta$ is hereditarily paracompact). Must X have a point-countable base?

D. Paracompactness and Generalizations

44. (Kaori Yamazaki, *Normality and collectionwise normality of product spaces*) Let X be a collectionwise normal space and Y a paracompact Σ-space (or a paracompact σ-space, or a M_3-space). Suppose $X \times Y$ is normal and countably paracompact. Then is $X \times Y$ collectionwise normal?

G. Mappings of Continua and Euclidean Spaces

32. (Joel Haywood, *A partial characterization of universal images of graphs using decompositions of trees*) If $f : G \to G'$ is a universal function, is it possible that G is a graph but not a tree? Show that if G' is a graph, then it is a tree.

33. (W. J. Charatonik and J. J. Charatonik, *Universality of weakly arc-preserving mappings*) Let $f : X \to Y$ be a surjective mapping between continua. Under what conditions about f and about Y the mapping f is universal? In particular, is f universal if f satisfies some conditions related to confluence

and Y is a) a dendrite, b) a dendroid, c) a λ-dendroid, d) a tree-like continuum having the fixed point property?

34. (W. J. Charatonik and J. J. Charatonik, *ibid.*) Does there exist an arcwise connected, unicoherent and one-dimensional continuum X and a confluent mapping from X onto a locally connected continuum Y which is not weakly arc-preserving?

35. (W. J. Charatonik and J. J. Charatonik, *ibid.*) For what continua X and Y a) is each confluent mapping $f : X \to Y$ weakly arc-preserving? b) is each weakly arc-preserving mapping $f : X \to Y$ weakly confluent?

36. (W. J. Charatonik and J. J. Charatonik, *ibid.*) Is every weakly arc-preserving mapping from a continuum onto a dendroid universal?

37. (W. J. Charatonik and J. J. Charatonik, *ibid.*) Is any confluent mapping from a continuum (from a dendroid) onto a dendroid universal?

33–37 Comments of the proposers. A mapping $f : X \to Y$ between continua is said to be *arc-preserving* provided that it is surjective, and for each arc $A \subset X$, its image $f(A)$ is either an arc or a point; it is *weakly arc-preserving* provided that there is an arcwise connected subcontinuum X' of X such that the restriction $f|X' : X' \to Y$ is arc-preserving. For more information concerning the subject see the proposers' article *Universality of weakly arc-preserving mappings*, this volume, pp. 123–154.

P. Products, Hyperspaces, Remainders, and Similar Constructions

43. (W. J. Charatonik) Let a mapping $f : X \to Y$ between continua X and Y be such that the induced mapping $C(f)$ is a near-homeomorphism (in particular, $C(X)$ and $C(Y)$ are homeomorphic). Does it imply that 2^f is a near-

homeomorphism? The same question, if $X = Y$.

44. (W. J. Charatonik) Let a mapping f between continua be such that the induced mapping 2^f is confluent. Does it imply that the induced mapping $C(f)$ is also confluent?

43–44 Comments of the proposer. For a given metric continuum X the symbols 2^X and $C(X)$ denote the hyperspaces of all nonempty closed subsets and of all nonempty subcontinua of X, respectively. Similarly, given a mapping $f : X \to Y$ between continua, the symbols 2^f and $C(f)$ denote the induced mappings. For more information on these concepts see the survey article by J. J. Charatonik, *Recent results on induced mappings between hyperspace of continua*, this volume, pp. 103–122 and the references therein.

R. Dimension Theory

21. (M. G. Charalambous, *A normal space Z with $\operatorname{ind} Z = 1$ no compactification of which has transfinite dimension*) Is there a perfectly normal space Y with $\operatorname{ind} Y = 1$ such that no Lindelöf (or even strongly paracompact) extension of Y has small transfinite inductive dimension?

S. Problems Closely Related to Set Theory

22. (Artur Hideyuki Tomita, *On the square of Wallace semigroups and topological free abelian groups*) Let κ be the least cardinal such that if G is a free abelian group endowed with a group topology, then G^κ is not countably compact. Under $\mathrm{MA}_{\mathrm{countable}}$, $\kappa > 1$, and in ZFC, $\kappa \leq \omega$. Find a better bound for κ or determine which cardinals κ may be. In particular, is it true that $\kappa > 1$ in ZFC? Is it consistent that $\kappa > 2$? Is ω the best upper bound for κ?

23. (Artur Hideyuki Tomita, *ibid.*) Let λ be the least cardinal such that if S is a both-sided cancellative semigroup which is not a group, endowed with a group topology, then S^λ

is not countably compact. Under $\mathrm{MA}_{\mathrm{countable}}$, $\lambda > 1$, and in ZFC, $\lambda < 2^{c}$. Find a better bound for λ or determine which cardinals λ may be. Is there a relation between κ and λ?

24. (Artur Hideyuki Tomita, *ibid.*) Is there (consistently) a free ultrafilter p over ω such that every p-compact group has a convergent sequence? Is it consistent that for every free ultrafilter p over ω there exists a p-compact group without non-trivial convergent sequences?

22–24 Comments of the proposer. Under $\mathrm{MA}_{\mathrm{countable}}$ there are 2^{c} many free ultrafilters p such that there exists for each of them a p-compact group without non-trivial convergent sequences. See Tomita and Watson, (expected title *Countable compactness in products of groups without non-trivial convergent sequences*).