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A DECOMPOSITION THEOREM FOR LOCALLY CONNECTED SUSLINIAN CONTINUA

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Abstract

This paper is an extension of early work by Simone, and later work by Dale Daniel. A continuum X is said to be Suslinian provided it does not contain uncountably many mutually exclusive nondegenerate subcontinua. In this paper we study the upper semi-continuous decomposition Y of a locally connected Suslinian continuum X, where Y is generated by the relation R such that xRy if and only if x and y belong to a subcontinuum g of X, where g is a subset of the closure of a countable set. We find that X/Y is a net-like continuous image of an arc, and are able to give necessary and sufficient conditions for X to be the continuous image of an arc, or to be monotonically normal. It is also shown that if a locally connected Suslinian continuum is not the continuous image of an arc, then there is a separable, locally connected, Suslinian continuum which is not the continuous image of an arc.

Definition. A continuum X is said to be Suslinian if there does not exist an uncountable family of mutually exclusive nondegenerate continua in X.

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Theorem 1. *Let X denote a locally connected Suslinian continuum. Let Y be a decomposition of X generated by the relation R such that xRy if and only if x and y belong to a subcontinuum g of X such that g is a subset of the closure of a countable set. Then, Y is an upper semi-continuous decomposition of X into continua such that X/Y is a net-like IOC.*

Theorem 2. *With X, R, Y as in Theorem 1, the following are equivalent:*

(a) *each $y \in Y$ is a metric continuum,*

(b) *X is an IOC,*

(c) *X is monotonically normal.*

Theorem 3. *With X as in Theorem 1, if X is not an IOC, then there is a separable, locally connected, Suslinian continuum which is not an IOC.*

In [9] Simone shows the following Theorem S. If X is a first countable, Suslinian, connected IOK and Y is a decomposition of X generated by the relation s such that xsy if and only if x and y lie in a metric subcontinuum of X, then

(1) Y is an upper semi-continuous decomposition of X into continua,

(2) each $y \in Y$ is a metric continuum,

(3) X/Y is a net-like IOC.

In [1] Daniel investigates a countably tight, locally connected, Suslinian continuum X such that every separable subset of X is hereditarily separable, and uses the relation r such that xry if and only if x and y belong to a separable subcontinuum of X. With these definitions he obtains theorems analogous to the Theorems 1 and 2 above.

Other related papers are by Simone [10] and Tymchatyn [13].

Some definitions are now in order. A compact Hausdorff space is called an *IOC* if it is the continuous image of an arc A and is called an *IOK* if it is the continuous image of a compact ordered space X. A continuum C is said to be *countably tight* provided that if t is a limit point of some subset A of C, then t is a limit point of some countable subset of A. A topological space X is said to be *monotonically normal* provided it is true that for each point x of X and open set U containing x there is an open set $H(x,U)$ such that

(1) $x \in H(x,U) \subset U$,

(2) if also U' is open and $U \subset U'$, then $H(x,U) \subset H(x,U')$, and

(3) if x and y are distinct points of X, then $H(x,X-y) \cap H(y,X-x) = \emptyset$.

A topological space X is said to be *connected imkleinen at the point* P provided it is true that if P is an element of an open set U, then the component of P in U contains an open set containing P. X is said to be *connected imkleinen* if X is connected imkleinen at each point. A continuum X is said to be *hereditarily locally connected* provided every subcontinuum of X is locally connected. A *cyclic element* of a locally connected continuum is a subcontinuum of X that is maximal with respect to the property of being a subcontinuum of X that is not separated by a point. A continuum X is said to be *net-like* provided each pair of points of X are separated by some finite set.

Throughout the remainder of the paper X, R, Y will be as stated in Theorem 1, and also for each $x \in X$ let $K_x = \{y \in X\colon\ xRy \text{ holds}\}$.

Lemma 1. *R is an equivalence relation.*

Proof. Straightforward. □

Lemma 2. *If $x \in X$, then K_x is a continuum.*

Proof. Clearly K_x is connected, so we need only show that K_x is closed. Suppose $y \in \overline{K}_x - K_x$. We define a well ordered sequence $C_{\alpha_1}, C_{\alpha_2}, \ldots$ of continua as follows.

Let U_{α_1} denote an open set containing y, where $x \notin \overline{U_{\alpha_1}}$. There is a point y_{α_1} of K_x, a continuum g_{α_1}, and a countable set S_{α_1} such that $x, y_{\alpha_1} \in g_{\alpha_1} \subset K_x$, $y_{\alpha_1} \in U_{\alpha_1}$ and $g_{\alpha_1} \subset \overline{S_{\alpha_1}}$. If g_{α_1} contains y, we have a contradiction, so let C_{α_1} denote the closure of the component of $g_{\alpha_1} \cap U_{\alpha_1}$ containing y_{α_1}.

Now suppose α_i is an ordinal $< \omega_1$ and that $U_\alpha, g_\alpha, y_\alpha, S_\alpha, C_\alpha$ have been defined for $\alpha < \alpha_1$.

Case 1. If y is a limit point of $g = \overline{\bigcup_{\alpha<\alpha_1} g_\alpha}$, then we let $S_g = \bigcup_{\alpha<\alpha_i} S_\alpha$ and note that $g \subset \overline{S}_g$ and that $y \in K_x$, a contradiction.

Case 2. $y \notin \overline{\bigcup_{\alpha<\alpha_1} g_\alpha}$. Let U_{α_i} denote an open set containing y and where $\overline{U}_{\alpha_i} \cap \overline{\bigcup_{\alpha<\alpha_i} g_\alpha} = \emptyset$. Let y_{α_i} denote a point of K_x in U_{α_i}, let g_{α_i} denote a subcontinuum of K_x containing x and y_{α_i}, let S_{α_i} denote a countable set such that $g_{\alpha_i} \subset \overline{S_{\alpha_i}}$, and let C_{α_i} denote the closure of the component of $g_{\alpha_i} \cap U_{\alpha_i}$ which contains y_{α_i}.

Since y was assumed to not be in K_x, then $\{C_\alpha, \alpha < \omega_1\}$is an uncountable collection of mutually exclusive nondegenerate continua lying in X, a contradiction. Therefore K_x is closed.□

Lemma 3. *For each $x \in X$, there is a countable set S_x such that $K_x \subset \overline{S}_x$.*

Proof. We define a well ordered sequence $C_{\alpha_1}, C_{\alpha_2}, \ldots$ of continua as follows. Let y_{α_1} denote a point of $K_x - \{x\}$ and let U_{α_1} denote an open set containing y_{α_1} such that $x \notin \overline{U}_{\alpha_1}$. Let g_{α_1} denote a subcontinuum of K_x containing x and y_{α_1}, let S_{α_1} denote a countable set such that $g_{\alpha_1} \subset \overline{S}_{\alpha_1}$, and let C_{α_1} denote the closure of the component of $g_{\alpha_1} \cap U_{\alpha_1}$ which contains y_{α_1}.

Now suppose that $\alpha_i < \omega_1$ and that $y_\alpha, g_\alpha, S_\alpha$, and C_α have been defined for $\alpha < \alpha_i$. If $K_x \subset \overline{\bigcup_{\alpha<\alpha_i} S_\alpha}$ we are done, so suppose $y_{\alpha_i} \in K_x - \overline{\bigcup_{\alpha<\alpha_i} S_{\alpha_i}}$. Let U_{α_i} denote an open set such that $y_{\alpha_i} \in U_{\alpha_i}$ and $\overline{U_{\alpha_i}} \cap \overline{\bigcup_{\alpha<\alpha_i} S_\alpha} = \emptyset$. Let g_{α_i} denote a subcontinuum of K_x containing x and y_{α_i} such that there is a countable set S_{α_i} such that $g_{\alpha_i} \subset \overline{S_{\alpha_i}}$. Finally, let C_{α_i} denote the closure of the component of $g_{\alpha_i} \cap U_{\alpha_i}$ containing y_{α_i}. Thus, we find that either K_x is a subset of the closure of a countable set, or that $\{C_\alpha\colon \alpha < \omega_1\}$ is an uncountable collection of mutually exclusive nondegenerate continua, a contradiction. □

Lemma 4. $Y = \{K_x\colon x \in X\}$ *is an upper semi-continuous decomposition of* X *into continua.*

Proof. Let $K_{x_0} \in Y$ and suppose $K_{x_0} \subset U$ open. Let $Q = \{K_x\colon K_x \in Y \text{ and } K_x \cap U \neq \emptyset \text{ and } K_x \cap (X-U) \neq \emptyset\}$. Since Q is a collection of mutually exclusive nondegenerate continua, then $Q = \{K_{x_1}, K_{x_2}, K_{x_3}, \ldots\}$.

Case 1. $\overline{\bigcup_{i>0} K_{x_i}} \cap K_{x_0} \neq \emptyset$. We note that some component of the limiting set L of $K_{x_1}, K_{x_2}, K_{x_3}, \ldots$ meets K_{x_0} and $X-U$. For suppose not. Then $L = R \cup S$, mutually separated, where $L \cap K_{x_0} \subset R$ and $L \cap (X-U) \subset S$. Let V and W denote disjoint open sets such that $R \subset V \subset U$ and $S \cup (X-U) \subset W$. Since infinitely many of the K_{x_i} meet $X - (V \cup W)$, then $L \cap (X - (V \cup W)) \neq \emptyset$, a contradiction. Thus, let C denote a component of L containing a point Z of K_{x_0} and a point of $X-U$. For each $i = 0, 1, 2, 3, \ldots$ let S_i denote a countable set such that $K_{x_i} \subset \overline{S_i}$. Now $K_{x_0} \cup C$ is a continuum and $(K_{x_0} \cup C) \subset \overline{\bigcup_{i\geq 0} S_i}$, so $(K_{x_0} \cup C) \subset K_{x_0}$, a contradiction.

Case 2. $\overline{\bigcup_{i>0} K_{x_i}} \cap K_{x_0} = \emptyset$. Let $V = U - \overline{\bigcup_{i>0} K_{x_i}}$. If $K_y \in Y$ and $K_y \cap V \neq \emptyset$, then $K_y \subset U$. Therefore, Y is upper semi-continuous. □

Definition. Let $K_1, K_2, K_3, \ldots$ denote the nondegenerate elements of Y.

Lemma 5. *If C is a countable set then every component of* $A = Cl\left(C \cup \left(\bigcup_{i\geq 1} K_i\right)\right)$ *is either a set K_i or a point.*

Proof. Suppose Q is a component of A which is neither a point nor a set K_i. For each j let C_j denote a countable set such that $K_j \subset \overline{C}_j$.

Since card. $Q \geq 2$ and $Q \subset Cl\left(C \cup \left(\bigcup_{j>0} C_j\right)\right)$, then Q is a subset of some K_j. Since Q is a component of A, then $K_j \subset Q$. Therefore $K_j = Q$ and we have a contradiction. □

Proof of Theorem 1.

If X/Y is hereditarily locally connected, then by [6] X/Y is the continuous image of an arc. Thus, assume X/Y is not hereditarily locally connected. Then there is a subcontinuum C of X/Y such that C is not connected imkleinen at some point p of C. Then, by [4] there exists a connected open set U in X/Y, a sequence (O_n) of connected open sets in X/Y containing p, a sequence (V_n) of mutually exclusive open sets in X/Y, and a sequence (G_n) of continua in X/Y such that

1) $U \supset \overline{O}_1 \supset O_1 \supset \overline{O}_2 \supset O_2 \supset \cdots$
2) $G_n \cap O_n \neq \emptyset$ and $G_n \cap O_{n+1} = \emptyset$ for each n,
3) each G_n is a component of $\overline{U} \cap C$ and $G_n \cap \partial U \neq \emptyset$ for all n, and
4) $G_n \cap G_m = \emptyset$ if $n \neq m$, and $G_n \subset V_n$ for all n.

Let h_0 denote the limiting set of $h_1, h_2, h_3, \ldots$ where h_n is a component of $G_n - O_1$ which intersects ∂O_1 and ∂U for each n. Let $s_n \in h_n \cap \partial O_1$ and $t_n \in h_n \cap \partial U$ for each n. By [7] some component of h_0 contains s and t, respectively, where $s \in L_s =$ limiting set of (s_n) and $t \in L_t =$ limiting set of (t_n), and if

V is a neighborhood of s and W is a neighborhood of t, then $(s_n, t_n) \in V \times W$ for infinitely many n.

Now suppose $\cup h_0 \subset \overline{\bigcup_{n\geq 1} K_n}$. Since $\overline{\bigcup_{n\geq 1} K_n}$ is a subset of the closure of a countable set then s, t would be subsets of the same element of Y, a contradiction. Therefore $\overline{\cup h_0} \not\subset \overline{\bigcup_{n\geq 1} K_n}$ and $\cup h_0 - \overline{\bigcup_{n\geq 1} K_n}$ contains a nondegenerate subcontinuum of X. In $X - \overline{\bigcup_{n\geq 1} K_n}$, the elements of Y are degenerate and also $X - \overline{\bigcup_{n\geq 1} K_n}$ is not hereditarily locally connected since the original construction can be repeated at some pont of h_0. So, we may assume without loss of generality that $\overline{\cup U} \subset X - \overline{\bigcup_{n\geq 1} K_n}$.

There exists a sequence (C_n) of finite covers of X by connected open sets such that

(1) each C_{n+1} is a star-refinement of C_n,

(2) for each n there are not two intersecting elements of C_n one of which intersects one of $\cup h_0, \cup h_1, \ldots, \cup h_n$ and the other of which intersects another one of $\cup h_0, \cup h_1, \ldots, \cup h_n$,

(3) for each n there are not two intersecting elements of C_n, one intersecting $\overline{\cup U}$ and the other $\overline{\bigcup_{n\geq 1} K_n}$.

Now $K = \{k\colon\ k \subset X, k = \bigcap_{n\geq 1} \text{star}(x, C_n)\}$ is a metric upper semi-continuous decomposition of X into continua. Let W denote the collection of nondegenerate elements of K. Since X/K is compact metric, $K - W$ is separable. Let D denote a countable dense subset of $K - W$. If every point of $\cup h_0$ were a limit point of D, then $\cup h_0$ would lie in a single element of Y, a contradiction. There is a point $q \in \cup h_0$ so that q is not a limit point of D. Thus, there is a connected open set Q containing q so that $\overline{Q} \cap D = \emptyset$. But $\overline{Q} = \bigcup_{w\in W} \{w \cap \overline{Q}\}$. Since no continuum is the union of a countable number greater than one of disjoint closed sets, there is a contradiction. Therefore X/Y is hereditarily locally connected and is the continuous image of an arc.

Now let A, B be distinct elements of X/Y and suppose no finite set separates A from B. By Treybig [12] there is a metric subcontinuum M of X/Y which contains A, B. There is a countable dense set $\{\{y_1\}, \{y_2\}, \{y_3\}, \ldots\}$ consisting of singleton elements of X/Y which is dense in M. If for each i, S_i is a countable set so that $K_i \subset \overline{S}_i$, then $\cup M$ is a subset of $Cl[\{y_1, y_2, y_3, \ldots\} \cup (\cup S_i)]$ and so $\cup M$ is a subset of a single element of Y, a contradiction.

This completes the proof of Theorem 1. □

Definition. From this point forward let $A = [a, b]$ denote a generalized arc and let $f\colon A \to X/Y$ denote a continuous onto map, and let $\phi\colon X \to X/Y$ denote the natural map.

Lemma 6. *Let C denote a countable subset of X. Then the space Z of components of $A = Cl(C \cup (\cup K_j))$ is a totally disconnected compact metric space which is a G_δ set in X/Y.*

Proof. By Lemma 5 each component of A is either a set K_j or a point, so the space of components of A is a closed subset of X/Y and is totally disconnected in X/Y.

Since X/Y is Suslinian, then by Mardesic [3] Z is a G_δ set in X/Y. Let $V_1, V_2, V_3, \ldots$ denote a sequence of open sets in X/Y so that $\overline{V}_1 \supset V_1 \supset \overline{V}_2 \supset V_2 \supset \cdots$ and $\bigcap_{i=1}^{\infty} V_i = \phi(A) = Z$, where each component of each V_i contains an element of Z. Since X/Y is locally connected, each V_i has only a finite number of components, so it is readily seen that $\{\phi(b \cap \cup Z)\colon b$ is a component of some $V_n\}$ forms a countable basis for Z. Thus, Z is metrizable. □

Lemma 7. *If $z \in X - \overline{\cup K_j}$ and V is a connected open neighborhood of z containing no point of $\overline{\cup K_j}$, then there is a finite subset F of V which separates z from $Bd\ V$.*

Proof. Let G denote the upper semi-continuous decomposition of X/Y consisting of the points $\{v\}$, for $v \in V$, together with the single closed set $\phi[X - V]$. Let α: $X/Y \to X/Y\Big/G$ denote the natural map. By Treybig [12] some finite subset $\alpha\phi(v_1), \ldots, \alpha\phi(v_n)$ of $\alpha\phi(V)$ separates $\alpha\phi(z)$ from $\alpha\phi(X - V)$, in $X/Y\Big/G$. Then $\{v_1, \ldots, v_n\}$ is the desired set. □

Proof of Theorem 2.

(1) (a) → (b). Suppose each $y \in Y$ is metrizable. Without loss of generality we may assume that no point separates X (i.e., X is the only cyclic element of X).

To show that X is an IOC, it suffices by [5] to show that if p, q, r are three points of X, then there is a metrizable T set containing p, q, r. By Lemma 6, assuming $C = \emptyset$, there is a sequence $V_1, V_2, V_3, \ldots$ of open sets in X/Y so that $\overline{V}_1 \supset V_1 \supset \overline{V}_2 \supset V_2 \supset \cdots$ and $\phi(\overline{\cup K_i}) = \bigcap_{i \geq 1} V_i = \bigcap_{i \geq 1} \overline{V}_i$. Thus, $\bigcap_{i \geq 1} (\cup V_i) = \overline{\bigcup_{i \geq 1} K_i}$.

With the aid of finite covers of the boundaries of the sets of the form $\cup V_i$ by open sets with finite boundaries (see Lemma 7) we find a sequence $W_1, W_2, W_3, \ldots$ of open sets such that

(1) $\overline{W}_1 \supset W_1 \supset \overline{W}_2 \supset W_2 \supset \cdots$,

(2) $\bigcap_{i \geq 1} W_i = \bigcap_{i \geq 1} \overline{W}_i = \overline{\cup K_j}$,

(3) each W_i has a finite boundary F_i, and only a finite number of components.

We let D denote a countable subset of X such that

1) $\{p, q, r\} \subset D$,

2) $\overline{D}$ contains each K_i.

We now apply the methods of Treybig [11, 12] to f: $A \to X/Y$ and the countable set $\phi(D)$ in X/Y to obtain a countable set $D' = \{K_1, K_2, K_3, \ldots \{c_1\}, \{c_2\}, \{c_3\}, \ldots\}$ such that

(1) $\phi(D) \subset D'$,

(2) each component of $X/Y - \overline{D'}$ has a two point boundary consisting of points not in $\phi(\overline{\cup K_i})$ (because of the sets F_i).

Now let $L = Cl(D \cup \{c_1, c_2, c_3, \ldots\})$ and use Lemma 6 to observe that the space of components Z' of L is metrizable. There exist countable sets of continuous functions $C_0, C_1, C_2, \ldots$ from L into $[0,1]$ such that

(1) if x, y belong to different components in Z', then x, y are separated by some function in C_0,

(2) if x, y are different points in some K_i, there is a function f in C_i which separates x, y.

The functions in $\bigcup_{i \geq 0} C_i$ allow us to embed L in the Hilbert cube, so L is metrizable. Therefore, X is an IOC.

(b) → (c) Suppose X is an IOC. By Heath, Lutzer, and Zenor [2], X is monotonically normal.

(c) → (a) Let $c \in C$. Then there is a countable set S so that $c \subset \overline{S}$. Since a monotonically normal space has the property that a separable set is hereditarily separable, then c is separable. By Rudin [8], c is an IOK. By Treybig [11] c is metrizable. □

Proof of Theorem 3.

Using X, R, Y as in Theorem 1, suppose that X is not an IOC. By Theorem 2 some K_i, say K_1, is not metrizable. For each i there exists a countable set C_i in X such that $K_i \subset \overline{C}_i$.

As in the proof of Theorem 2, there is a sequence $W_1, W_2, \ldots$ of open sets such that

(1) $\overline{W}_1 \supset W_1 \supset \overline{W}_2 \supset \overline{W}_2 \supset \cdots$,

(2) $\bigcap_{n \geq 1} \overline{W}_i = \bigcap_{i \geq 1} W_i = \overline{\bigcap_{j \geq 1} K_j}$, and,

(3) each W_i has a finite boundary F_i, and has only a finite number of components.

Let D denote the countable set $\{K_1, K_2, K_3, \ldots\} \cup \phi(\cup C_i) \cup \phi(\cup F_i)$. By the methods used in [11, 12] there is a countable set D' in X/Y such that

(1) $D \subset D'$,

(2) each component of $X/Y - \overline{D'}$ has a one or two point boundary, and

(3) no element K_j is a boundary element of a component of $X/Y - \overline{D'}$ (because of the sets F_i).

We now let X' denote $X - \cup O_1$ where $O_1 = \{$subsets J of X such that $\phi(J) = T$, where T is a component of X/Y—$\overline{D'}$ with a one point boundary$\}$. Clearly, X' is a locally connected Suslinian continuum which is not an IOC.

Now let $O_2 = \{$subsets J of X such that $\phi(J) = T$, where T is a component of $X/Y - \overline{D'}$ with a two point boundary$\}$. Let $Bd(J) = \{a_J, b_J\}$, where $Bd.\ T = \{\{a_J\}, \{b_J\}\}$. Since [3] implies that X/Y is first countable at each of $\{a_J\}$ and $\{b_J\}$, then X' is first countable at each of a_J and b_J and there is an onto continuous map f_J: $\overline{J} \to [0,1]$ such that $f_J(a_J) = 0$, $f_J(b_J) = 1$, and $f_J^{-1}(0) = a_J$ and $f_J^{-1}(1) = b_J$.

We now consider the upper semi-continuous decomposition G of X' defined by $G = \{\{x\}$: $x \in X' - \cup O_2\} \cup \{f_J^{-1}(t)$: $J \in O_2$ and $0 < t < 1\}$. It is straightforward to show that X'/G is a locally connected Suslinian continuum, but we now wish to show that X'/G is separable, but not an IOK. Let α: $X' \to X'/G$ denote the natural map.

Now O_2 is countable since X' is Suslinian, so for each J in O_2 let $C_J = \{f_J^{-1}(q)$: q is a rational number in [0,1]$\}$. Also for each i let $C_i' = \alpha(C_i - (\cup O_1) \cup (\cup O_2))$ and let $F_i' = \alpha(F_i)$. We show that the countable set $S = (\cup C_J) \cup \left(\bigcup_{i \geq 1} C_i'\right) \cup \left(\bigcup_{i \geq 1} F_i'\right)$ is dense in X/G.

Let $\{x\} \in X/G - \overline{\cup C_J}$, and let $x \in V$ where $\alpha(V)$ is a connected open set in $X/G - \overline{\cup C_J}$.

Case 1. $x \in K_i$ for some i. Then there is a point t of C_i in V, so $\{t\} \in \alpha(V)$, where $\{t\} \in C_i'$.

Case 2. $x \in \overline{\cup K_i}$. Then $\{x\} \in \overline{\cup C_i'}$.

Case 3. $\{x\} \in X'/G - (\alpha(\overline{\cup K_i}) \cup (\overline{\cup C_J}))$.

Then there is a nondegenerate subcontinuum Z of X'/G lying in $X'/G - (\alpha(\overline{\cup K_i}) \cup (\overline{\cup C_J}))$. But Z would intersect some component C_J since Z cannot be a subset of the union of singleton elements of $\overline{D_i'}$ a contradiction. Since $\cup C_J$ is dense in $\overline{\cup C_J}$, it is clear that S is dense in X'/G.

Finally, if we pick R' and Y' for X'/G analogous to R and Y for X, then $\alpha(K_1)$ is a subset of a single element y' of Y'. Since y' is not metric, X'/G is not an IOC. □

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