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ON THE EMBEDDING AND DEVELOPABILITY OF MAPPING SPACES WITH COMPACT OPEN TOPOLOGY

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ABSTRACT. We investigate the relation of mapping spaces with compact open topology and hyperspaces of compact subsets with finite topology. Using one of the results, we show the Moore spaces with a regular G_δ-diagonal are hereditary to these mapping spaces.

1. INTRODUCTION.

All spaces are assumed to be regular T_2. For a space X, we denote by $\tau(X)$ the topology of X. Throughout this paper, letter $\mathbb{N}$ means the set of all positive integers. For families $\mathcal{U}$, $\mathcal{V}$ of subsets of X, $\mathcal{U} < \mathcal{V}$ means that for each $U \in \mathcal{U}$, there exists $V \in \mathcal{V}$ such that $U \subset V$. Let $\mathcal{K}(X)$ be the set of all non-empty compact subsets and for the topology of $\mathcal{K}(X)$, we use here Vietoris topology, which has the base consisting of all subsets of the form

$$\langle U_1, \cdots, U_k\rangle = \{K \in \mathcal{K}(X) | K \subset \bigcup\{U_i | i = 1, \cdots, k\} \text{ and } K \cap U_i \neq \emptyset \text{ for each } i\}.$$

where $U_1, \cdots, U_k \in \tau(X)$, $k \in \mathbb{N}$. For brevity, we use frequently the notation $\langle \mathcal{U}\rangle$ or $\langle U_i | i = 1, \cdots, k\rangle$ in place of

$\langle U_1, \cdots, U_k \rangle$, where $\mathcal{U} = \{U_1, \cdots, U_k\}$. As known already, if X is regular T_2, then so is $\mathcal{K}(X)$. As for the fundamental properties of $\mathcal{K}(X)$, refer to [M]. For spaces X, Y, let $C(X,Y)$ be the set of all continuous mappings of X into Y. As the topology of $C(X,Y)$, we accept the compact open topology, which has the base consisting of subsets of the form

$$W(K_1, \cdots, K_n; O_1, \cdots, O_n) = \{f \in C(X,Y) | f(K_i) \subset O_i \text{ for each } i\},$$

where $K_i \in \mathcal{K}(X)$ and $O_i \in \tau(Y)$ for each i. This space is written as $C_k(X,Y)$.

In the first part, we investigate the relation between mapping spaces and hyperspaces. We show that for a space X, $C_k(X,Y)$ is embedded into the product spaces of hyperspaces. This embedding is shown to have some additional properties for special spaces X. The main result is used in the second part.

Next, we consider the classical problem on heredity of topological properties: Let $\mathcal{P}$ be a class of spaces and let X be a compact or hemicompact space. If $Y \in \mathcal{P}$, then does $C_k(X,Y) \in \mathcal{P}$? Until now, we have some results for $\mathcal{P}$ of metric spaces [A], $\aleph_0$-spaces [M] or paracompact $\aleph$-spaces [O] etc. But we do not know whether Moore spaces are hereditary to $C_k(X,Y)$ when X is a compact space. Here, we show that this is the case for Moore spaces with a regular G_δ-diagonal.

As for undefined terms such us G_δ-diagonals, wΔ-spaces, etc, refer to [G].

2. The embedding of $C_k(X,Y)$ into hyperspaces.

Theorem 2.1. *Let $\mathcal{K}$ be a compact cover of a space X such that $\mathcal{K}(X) < \mathcal{K}$. Then for a space Y,*

$$C_k(X,Y) \hookrightarrow \prod \{\mathcal{K}(K \times Y) | K \in \mathcal{K}\}.$$

Proof: As in the proof of (g) implies (a) in [MN, Theorem 3.2], the restriction map embeds $C_k(X,Y)$ into $\prod\{C_k(K,Y) | K \in \mathcal{K}\}$. As is well known (see, for example, exercise 3.12.27(j) in

[E]), for compact K the map G taking a function in $C_k(K,Y)$ to its graph in $\mathcal{K}(K \times Y)$ is an embedding. Combining these embeddings gives the claim.

We remark that by virtue of the above theorem, when X is compact, $C_k(X,Y)$ is embedded into $\mathcal{K}(X \times Y)$ with the embedding $G : C_k(X,Y) \to \mathcal{K}(X \times Y)$ such that for each $f \in C_k(X,Y)$, $G(f)$ is the graph of f.

A space X is called *hemicompact* [A] if there exists a countable compact cover $\mathcal{K}$ of X such that $\mathcal{K}(X) < \mathcal{K}$.

Corollary 2.2. *If X is a hemicompact space, then for a space Y $C_k(X,Y) \hookrightarrow \prod\{\mathcal{K}(K_i \times Y)|i \in \mathbb{N}\}$, where $\{K_i|i \in \mathbb{N}\} \subset \mathcal{K}(X)$.*

In the embedding theorem, we do not have any information on what kind of a subset of $\mathcal{K}(X \times Y)$ $\varphi(C_k(X,Y))$ is. The next two theorems give it for compact spaces X.

Theorem 2.3. *If X is a compact metrizable space and Y has a G_δ-diagonal, then $C_k(X,Y)$ is homeomorphic to a G_δ-set of $\mathcal{K}(X \times Y)$.*

Proof: The projection map, π_X of $X \times Y$ onto X extends to a continuous map of $\mathcal{K}(X \times Y)$ onto $\mathcal{K}(X)$. Since $\mathcal{K}(X)$ is metrizable, the point X in $\mathcal{K}(X)$ is a G_δ, and so $\mathcal{K}_0 = \{L \in \mathcal{K}(X \times Y)|\pi_X(L) = X\}$ is a G_δ in $\mathcal{K}(X \times Y)$.

In the notation of the paragraph preceding Lemma 3.3 for a G_δ-diagonal sequence $\{\mathcal{U}_n|n \in \mathbb{N}\}$ for Y, let $W_n = \bigcup \mathcal{W}[n]$. Then W_n is an open subset of $\mathcal{K}(X \times Y)$, and it is easy to check that $\mathcal{K}_0 \cap \bigcap_{n \in \mathbb{N}} W_n = G(C_k(X,Y))$. Thus $G(C_k(X,Y))$, a G_δ subset of $\mathcal{K}_0$ which is a G_δ subset of $\mathcal{K}(X \times Y)$, is also a G_δ subset

We state the definition of being equicontinuous of $C_k(X,Y)$.

Let $\mathcal{F} \subset C(X,Y)$, where X is a space and Y is a uniform space with the uniformity $\mu = \{\mathcal{U}_\alpha|\alpha \in A\}$. If for each $\alpha \in A$ and each $p \in X$, there exists a neighborhood $N(p)$ of p such that

$$f(N(p)) \subset \mathrm{S}(f(p), \mathcal{U}_\alpha)$$

for each $f \in \mathcal{F}$, then $\mathcal{F}$ is called *equicontinuous*, [N, p. 282].

Theorem 2.4. *Let X be a compact space and Y be a uniform space. If $C(X,Y)$ is equicontinuous, then $G(C_k(X,Y))$ is a closed subspace of $\mathcal{K}(X \times Y)$, i.e., $C_k(X,Y)$ is embedded into a closed subspace of $\mathcal{K}(X \times Y)$.*

Proof: We show that $G(C_k(X,Y))$ is closed in $\mathcal{K}(X \times Y)$. Take $L \in \mathcal{K}(X \times Y) \backslash G(C_k(X,Y))$. Suppose $|L \cap (\{x\} \times Y)| \geq 2$ for some $x \in X$. Take $\langle x, y_1 \rangle$, $\langle x, y_2 \rangle \in L$ with $y_1 \neq y_2$. Let $\mu = \{\mathcal{U}_\alpha | \alpha \in A\}$ be the uniformity of Y compatible with Y. Then there exists $\mathcal{U} \in \mu$ such that $y_1 \notin \mathrm{S}(y_2, \mathcal{U})$ and let $\mathcal{V}$ be an open cover of Y such that $\mathcal{V} \in \mu$ and $\mathcal{V}^{**} < \mathcal{U}$. For this $\mathcal{V}$, there exists an open neighborhood $N(x)$ of x in X such that $f(N(x)) \subset \mathrm{S}(f(x), \mathcal{V})$ for each $f \in C(X,Y)$. Take V_1, $V_2 \in \mathcal{V}$ such that $y_1 \in V_1$, $y_2 \in V_2$. Then it is easy to see

$$\hat{O} = \langle N(x) \times V_1, N(x) \times V_2, X \times Y \rangle$$

is an open neighborhood of L in $\mathcal{K}(X \times Y)$ such that $\hat{O} \cap G(C_k(X,Y)) = \emptyset$. Next, suppose $|L \cap (\{x\} \times Y)| \leq 1$ for each $x \in X$. If there exists $x \in X$ such that $L \cap (\{x\} \times Y) = \emptyset$. Then it is easy to see that $\langle (X \backslash \{x\}) \times Y \rangle$ is an open neighborhood of L in $\mathcal{K}(X \times Y)$ missing $G(C_k(X,Y))$. For the last case, we suppose $L = \{\langle x, b(x) \rangle | x \in X\}$. Since $L \notin G(C_k(X,Y))$, the correspondence b is not continuous. This means that for some $A \subset X$ there exists $y \in b(\overline{A}) \backslash \overline{b(A)}$. Let $y = b(x)$ with $x \in \overline{A}$. There exists $\mathcal{U} \in \mu$ and an open cover $\mathcal{V} \in \mu$ such that $\mathrm{S}(y, \mathcal{U}) \cap b(A) = \emptyset$ and $\mathcal{V}^{**} < \mathcal{U}$. Since $C_k(X,Y)$ is equicontinuous, there exists an open neighborhood $N(x)$ of x in X such that $f(N(x)) \subset \mathrm{S}(f(x), \mathcal{V})$. Take V, $V' \in \mathcal{V}$ such that $y \in V$, $b(x_0) \in V'$, where $x_0 \in N(x) \cap A$. Then it is easy to see that $\langle N(x) \times V, N(x) \times V', X \times Y \rangle$ is an open neighborhood of L in $\mathcal{K}(X \times Y)$ missing $G(C_k(X,Y))$. This completes the proof.

3. Mapping spaces and Moore spaces.

Let us recall the definition of a regular G_δ-diagonal: A space X has a *regular G_δ-diagonal* if the diagonal set of $X \times X$ is a regular G_δ-set, and equivalently, if there exists a sequence $\{\mathcal{U}(n) | n \in \mathbb{N}\}$ of open covers of X such that if $x \neq y$, x, $y \in X$, then there exists $n \in \mathbb{N}$ and open neighborhoods O, O' of x, y in X, respectively, such that $\mathrm{S}(O, \mathcal{U}(n)) \cap O' = \emptyset$, [Z, Theorem 1]. In this characterization, we can assume $\mathcal{U}(n+1) < \mathcal{U}(n)$, $n \in \mathbb{N}$, and this is assumed in the sequel without any specification.

Theorem 3.1. *If X is a compact space and Y has a regular G_δ-diagonal (G_δ-diagonal, G_δ^*-diagonal), then $C_k(X,Y)$ has a regular G_δ-diagonal (G_δ-diagonal, G_δ^*-diagonal, respectively).*

Proof: We show the case of a regular G_δ-diagonal and the others are the same. By the characterization, there exists a sequence $\{\mathcal{U}(n) | n \in \mathbb{N}\}$ of open covers of Y such that if $y \neq y'$, y, $y' \in Y$, then there exists $n \in \mathbb{N}$ and open neighborhoods O, O' of y, y' in Y, respectively, such that $\mathrm{S}(O, \mathcal{U}(n)) \cap O' = \emptyset$. We construct a sequence $\{\mathcal{W}(n) | n \in \mathbb{N}\}$ of open covers of $C_k(X,Y)$ by the following method $(*)$ which is used later frequently:

$(*)$ Let $n \in \mathbb{N}$ and $\{\delta = (\mathcal{K}(\delta), \mathcal{U}(\delta)) | \delta \in \Delta(n)\}$ be the totality of pairs of subfamilies $\mathcal{K}(\delta)$, $\mathcal{U}(\delta)$ of $\mathcal{K}(X)$, $\mathcal{U}(n)$, respectively, such that $\mathcal{K}(\delta) = \{K_1, \cdots, K_t\}$ is a finite cover of X and $\mathcal{U}(\delta) = \{U_1, \cdots, U_t\}$. For each $\delta \in \Delta(n)$, let

$$W(\delta) = W(K_1, \cdots, K_t; U_1, \cdots, U_t)$$

and $\mathcal{W}(n) = \{W(\delta) | \delta \in \Delta(n)\}$.

Since X is compact, for each $f \in C_k(X,Y)$ and $n \in \mathbb{N}$, we can easily find $\delta \in \Delta(n)$ such that $f \in W(\delta)$. Thus each $\mathcal{W}(n)$ is an open cover of $C_k(X,Y)$. Suppose $f \neq g$, f, $g \in C(X,Y)$. Then $f(x_0) \neq g(x_0)$ for some x_0. By the property of $\{\mathcal{U}(n)\}$, there exists $n \in \mathbb{N}$ and open neighborhoods O, O' of

$f(x_0)$, $g(x_0)$ in Y, respectively, such that $\mathrm{S}(O,\mathcal{U}(n)) \cap O' = \emptyset$. It is easy to check that

$$\mathrm{S}(W(\{x_0\};O),\mathcal{W}(n)) \cap W(\{x_0\};O') = \emptyset.$$

Hence by the characterization, $C_k(X,Y)$ has a regular G_δ-diagonal.

Corollary 3.2. *Let X be a compact space. If $\{\mathcal{U}(n)|n \in \mathbb{N}\}$ is a normal sequence of open covers of Y, then $\{\mathcal{W}(n)|n \in \mathbb{N}\}$, defined by the same method as (*) above, is also a normal sequence of open covers of $C_k(X,Y)$.*

Proof: We show $\mathcal{W}(n+1)^* < \mathcal{W}(n)$ under the condition $\mathcal{U}(n+1)^* < \mathcal{U}(n)$. Suppose $W(\delta) \cap W(\delta') \neq \emptyset$, δ, $\delta' \in \Delta(n+1)$, where

$$\mathcal{K}(\delta) = \{K_1,\cdots,K_s\},\ \mathcal{U}(\delta) = \{U_1,\cdots,U_s\},$$

$$\mathcal{K}(\delta') = \{L_1,\cdots,L_t\},\ \mathcal{U}(\delta') = \{V_1,\cdots,V_t\},$$

$$\mathcal{U}(\delta) \cup \mathcal{U}(\delta') \subset \mathcal{U}(n+1).$$

For each i, take $U_i' \in \mathcal{U}(n)$ such that $\mathrm{S}(U_i,\mathcal{U}(n+1)) \subset U_i'$. Let $\delta^* = (\mathcal{K}(\delta),\{U_1',\cdots,U_s'\}) \in \Delta(n)$. Then we can show $W(\delta') \subset W(\delta^*)$. Indeed, if $f \in W(\delta')$, then $f(L_j) \subset V_j$ for each $j = 1,\cdots,t$. Since $\mathcal{K}(\delta')$ covers X, for each $K_i \in \mathcal{K}(\delta)$, let $N(i) = \{j|L_j \cap K_i \neq \emptyset, j = 1,\cdots,s\}$, which implies

$$f(K_i) \subset \bigcup\{f(L_j)|j \in N(i)\} \subset \bigcup\{V_j|j \in N(i)\} \subset U_i'.$$

Therefore we have $f \in W(\delta^*)$. Hence we have $\mathcal{W}(n+1)^* < \mathcal{W}(n)$.

For each $\delta = (\mathcal{K}(\delta),\mathcal{U}(\delta)) \in \Delta(n)$, $n \in \mathbb{N}$, we define

$$W[\delta] = \bigcap\{\langle X \times Y \backslash (K_i \times (Y \backslash U_i))\rangle | K_i \in \mathcal{K}(\delta)\}.$$

Then obviously $W[\delta]$ is an open subset of $\mathcal{K}(X \times Y)$ such that $G(W(\delta)) = W[\delta] \cap G(C_k(X,Y))$. For each n, $\mathcal{W}[n] = \{W[\delta]|\delta \in \Delta(n)\}$ is an open cover of $G(C_k(X,Y))$ in $\mathcal{K}(X \times Y)$.

Lemma 3.3. *Let X be a compact space and let Y have a regular G_δ-diagonal. Then there exists a closed subspace $\mathcal{K}_0$ of $\mathcal{K}(X \times Y)$ containing $G(C_k(X,Y))$ such that if $f \in C(X,Y)$, $L \in \mathcal{K}_0$ with $L \neq G(f)$, then there exists $n \in \mathbb{N}$ such that $L \notin \mathrm{S}(G(f), \mathcal{W}[n])^-$.*

Proof: Let

$$\mathcal{K}_0 = \{L \in \mathcal{K}(X \times Y) | \pi_X(L) = X\},$$

where $\pi_X : X \times Y \to X$ is the projection. As easily checked, $\mathcal{K}_0$ is a closed subspace of $\mathcal{K}(X \times Y)$. We show that $\mathcal{K}_0$, $\{\mathcal{W}[n]\}$ have the property. Let $G(f) \in G(C_k(X,Y))$, $L \in \mathcal{K}_0$ with $G(f) \neq L$. Then there exists $\langle x, y\rangle \in L \backslash G(f)$. For the first case, suppose $y \notin f(X)$. Using compactness of $f(X)$ and the property of $\{\mathcal{U}(n)\}$, we can take $n \in \mathbb{N}$ and an open neighborhood O of y in Y such that $\mathrm{S}(f(X), \mathcal{U}(n)) \cap O = \emptyset$. Then it is easy to see that $\langle X \times Y, X \times O\rangle$ is an open neighborhood of L in $\mathcal{K}(X \times Y)$ such that

$$\mathrm{S}(G(f), \mathcal{W}[n]) \cap \langle X \times Y, X \times O\rangle = \emptyset.$$

For the second case, suppose $y \in f(X)$. Then $y \neq f(x)$. There exists disjoint open neighborhoods O, O' of $f(x)$, y in Y, respectively. We take the open neighborhood $\langle P_1 \times Q_1, P_2 \times Q_2\rangle$ of $G(f)$ in $\mathcal{K}(X \times Y)$ as follows:

(1)

$$Q_1 = Y \backslash \{y\},\ P_1 = f^{-1}(Q_1) \text{ and } P_2 \times Q_2 = f^{-1}(O') \times O'.$$

Since X is compact, there exists a closed cover $\{F_1, F_2\}$ of X such that $\emptyset \neq F_i \subset P_i$ for each $i = 1, 2$. By the property of $\{\mathcal{U}(n)\}$, for F_1, there exists $n_0 \in \mathbb{N}$ and an open neighborhood $V(y)$ of y in Y such that

$$\mathrm{S}(f(F_1), \mathcal{U}(n_0)) \cap V(y) = \emptyset. \qquad (2)$$

By virtue of (1) and (2), we can easily show the following:

$$(3)\quad \begin{array}{l} \text{If } G(f) \in W[\delta], \text{ where } \delta = (\mathcal{K}(\delta), \mathcal{U}(\delta)) \in \Delta(n_0), \\ \mathcal{K}(\delta) = \{K_1, \cdots, K_s\}, \\ \mathcal{U}(\delta) = \{U_1, \cdots, U_s\}, \text{ then for each } i = 1, \cdots, s \\ \qquad (f^{-1}(O) \times V(y)) \cap (K_i \times U_i) = \emptyset. \end{array}$$

From (3), it follows that $\langle f^{-1}(O) \times V(y), X \times Y \rangle$ is an open neighborhood of L in $\mathcal{K}(X \times Y)$ missing $\mathrm{S}(G(f), \mathcal{W}[n_0])$. Hence we have $L \notin \mathrm{S}(G(f), \mathcal{W}[n_0])^-$.

Lemma 3.4. *Let X' be a subspace of a $w\Delta$-space X and suppose that there exists a sequence $\{\mathcal{U}(n) | n \in \mathbb{N}\}$ of open covers of X' in X such that for each $x \in X'$, $y \in X$ with $x \neq y$, there exists $n \in \mathbb{N}$ such that $y \notin \mathrm{S}(x, \mathcal{U}(n))^-$. Then X' is a developable space.*

Proof: Let $\{\mathcal{U}'(n) | n \in \mathbb{N}\}$ be a wΔ-sequence for X. Let $\mathcal{V}(n) = (\mathcal{U}(n) \wedge \mathcal{U}'(n)) | X'$, $n \in \mathbb{N}$. Without loss of generality, we can assume $\mathcal{V}(n+1) < \mathcal{V}(n)$ for each n. To see that $\{\mathcal{V}(n)\}$ forms a development for X', let $p \in O \in \tau(X')$. Assume that for each n there exists $p_n \in \mathrm{S}(p, \mathcal{V}(n)) \setminus O$. Then $\{p_n\}$ has a cluster point p'. But by the property of $\{\mathcal{U}(n)\}$, we have $p = p'$, a contradiction. Hence $\mathrm{S}(p, \mathcal{V}(n)) \subset O$ for some n.

Theorem 3.5. *Let X be a hemicompact space. Then Y is a Moore space with a regular G_δ-diagonal if and only if so is $C_k(X, Y)$.*

Proof: If part follows easily from the fact that Moore spaces and regular G_δ-diagonals are hereditary and the fact $Y \hookrightarrow C_k(X, Y)$. Only if part: Since Moore spaces and regular G_δ-diagonals are countably productive and hereditary, by Corollary 2.2, it suffices to show it for a compact space X. Suppose that Y is a Moore space with a regular G_δ-diagonal and that X is a compact space. Since by Theorem 2.1 $C_k(X, Y)$ has a regular G_δ-diagonal, it suffices to show that $C_k(X, Y)$ is a Moore space. By Lemma 3.3, there exist a closed subspace $\mathcal{K}_0$ of $\mathcal{K}(X \times Y)$ containing $G(C_k(X, Y))$ and a sequence $\{\mathcal{W}[n] | n \in \mathbb{N}\}$ of open covers of $G(C_k(X, Y))$ in $\mathcal{K}(X \times Y)$ such that for each $G(f) \in G(C_k(X,Y))$, $L \in \mathcal{K}_0$ with $G(f) \neq L$, there exists $n \in \mathbb{N}$ such that $L \notin \mathrm{S}(G(f), \mathcal{W}[n])^-$. Let $\mathcal{U}(n) = \mathcal{W}[n]|\mathcal{K}_0$, $n \in \mathbb{N}$. By [SM, Theorem 2.2], there exists a perfect mapping of $\mathcal{K}(X \times Y)$ onto $\mathcal{K}(Y)$ because $\mathcal{K}(X)$ is compact. By [Mi], $\mathcal{K}(Y)$ is a Moore space. Thus $\mathcal{K}_0$ is a wΔ-space. Using Lemma 3.4 with $\mathcal{K}_0$ and $\{\mathcal{U}(n)\}$, $X' = G(C_k(X,Y))$ (and hence $C_k(X,Y)$) is a Moore space.

Exercising our discussion used in the proof of Lemma 3.3, we can settle the following proposition, the result of which is well known as the Arens theorem [A, Theorem 7]. But this is the "topological" version of his proof.

Proposition 3.6. *If X is a compact space and Y is a metrizable space, then $C_k(X,Y)$ is metrizable.*

Proof: Let $\{\mathcal{U}(n)|n \in \mathbb{N}\}$ be a strong development for Y [E, Theorem 5.4.2] such that $\mathcal{U}(n+1)^* < \mathcal{U}(n)$, $n \in \mathbb{N}$. Then by Corollary 3.2, $\mathcal{W}(n+1)^* < \mathcal{W}(n)$, $n \in \mathbb{N}$. So, for the metrizability of $C_k(X,Y)$ it suffices to show that $\{\mathcal{W}(n)\}$ is a development for $C_k(X,Y)$. Let $f \in W(K_1,\cdots,K_s;O_1,\cdots,O_s)$, where $K_i \in \mathcal{K}(X)$ and $O_i \in \tau(Y)$ for each i. Since $\{\mathcal{U}(n)\}$ is a strong development, for each i there exists $n(i) \in \mathbb{N}$ such that $\mathrm{S}(f(K_i),\mathcal{U}(n(i))) \subset O_i$. Let $n = \max\{n(i)|i = 1,\cdots,s\}$. For this n, we can easily show $\mathrm{S}(f,\mathcal{W}(n)) \subset W(K_1,\cdots,K_s; O_1,\cdots,O_s)$.

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