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# ON PRESERVATION OF FIBRANTS UNDER SHAPE FIBRATIONS

ALEXANDER BYKOV[1]

Abstract. One can show [BZ] that a shape fibration of compacta has a fibrant extension which is a Hurewicz fibration. This implies simple proofs for some results already well known for shape fibrations and also some new results. In this paper it is shown that a shape fibration, which is a strong shape domination, preserves fibrants and approximate polyhedra.

## 0. Introduction

Let $\mathcal{CM}$ be the category of compact metric spaces and let $h\mathcal{CM}$, $ssh\mathcal{CM}$ be the corresponding homotopy and strong shape categories. A morphism of the strong shape category $ssh\mathcal{CM}$ is called a strong shape domination (ssh-domination) if it has a right inverse.

A map $p$ of metric compacta will be also called ssh-domination if its image under the composition of canonical functors

$$\mathcal{CM} \to h\mathcal{CM} \to ssh\mathcal{CM}$$

is of that kind, i.e. ssh-domination. Together with the above mentioned categories we shall deal with the category $\mathcal{M}$ of metric spaces and continuous maps and with the category $Map\mathcal{M}$ of maps of $\mathcal{M}$.

We utilize the approach to the strong shape theory introduced by F. Cathey in [Ca1] and [Ca2] and we use his terminology. Shape fibrations here are in the sense of S. Mardešić and T.B. Rushing

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[MR78]. The present paper can be regarded as continuation of [BZ] and it is inspired by [CMT].

The main result is Theorem 2.1. It says that the image of a compact fibrant space under a map, which is both a shape fibration and an ssh-domination, is fibrant. The proof of this Theorem is based on the "fibrant" representation of shape fibrations given in Theorem 1.4.

## 1. Strong shape theory and shape fibrations

In this section we give all necessary definitions and results for convenience of the reader, though some knowledge of shape theory, corresponding to [MS82], is supposed.

The notion of shape fibration was introduced in [MR78] for metric compacta, and afterwards it was generalized for arbitrary spaces in [M]. The definition of a shape fibration uses the property AHLP applied to ANR-resolutions of maps. We shall need only resolutions for compact metric spaces and for maps of compact metric spaces. Throughout the paper we deal only with metric spaces.

Let $\underline{E} = \{E_i, q_i^j\}$ be an inverse sequence of ANR-spaces (not necessarily compact) and let $E$ be a compactum. A morphism $\underline{q} = \{q_i\} : E \to \underline{E}$ (in pro-$\mathcal{M}$) is *ANR-resolution* of $E$ iff $(E, \underline{q}) = \varprojlim \underline{E}$ and the following condition holds: for every $i$ and any neighborhood $U$ of $q_i(E)$ in $E_i$ there exists $j \geq i$ such that $g_i^j(E_j) \subseteq U$.

Let $\underline{p} = \{p_i\} : \underline{E} \to \underline{B}$, where $p_i : E_i \to B_i$, be a level map of inverse sequences $\underline{E} = \{E_i, q_i^j\}$ and $\underline{B} = \{B_i, r_i^j\}$. It can be regarded as an inverse sequence $\underline{p} = \{p_i, (q_i^j, r_i^j)\}$ in pro-Map$\mathcal{M}$. Let $p : E \to B$ be a map of compact spaces. A morphism $(\underline{q}, \underline{r}) : p \to \underline{p}$ is *ANR-resolution* of $\underline{p}$ iff $\underline{q}$ and $\underline{r}$ are ANR-resolutions of $E$ and $B$ respectively.

In case of maps of metric compacta (more generally, in case of proper maps of metric spaces), shape fibrations can be described using the more simple lifting property HLP. A map $p : E \to B$ of metric compacta is a *shape fibration* if it admits an ANR-resolution $(\underline{q}, \underline{r}) : p \to \underline{p}$ such that for any space $X$, each map $h : X \to E_{i+1}$ and each homotopy $H : X \times I \to B_{i+1}$ with $p_{i+1} \circ h = H_0$, there exists a homotopy $\tilde{H} : X \times I \to E_i$ satisfying $\tilde{H}_0 = q_i^{i+1} \circ h$ and $p_i \circ \tilde{H} = r_i^{i+1} \circ H$.

The lifting property can be even more simplified utilizing the regularity of HLP and the fact that each map $p_i$ is the composition of an SDR-map $E_i \to coCyl(p_i)$ and a Hurewicz fibration $coCyl(p_i) \to B_i$, where *SDR-map* is a map, which embeds one space into another as a strong deformation retract.

**Theorem 1.1.** *A map $p : E \to B$ of compact metric space is a shape fibration iff $p$ admits an ANR-resolution $p \to \underline{p}$, $\underline{p} = \{p_i\}$, such that each $p_i$ is a Hurewicz fibration.*

(For a proof, see [BZ],Th.5,p.203).

A closed subset $A$ of a space $X$ is a *shape strong deformation retract* of $X$ iff there exists an AR-space $M$ and a closed embedding $\alpha : X \hookrightarrow M$ such that the following condition holds: for every pair of neighborhoods $(U, V)$ of $(\alpha(X), \alpha(A))$ in $M$ there is homotopy $H : X \times I \to M \ rel.A$ with $H_0 = \alpha$, $ImH \subseteq U$ and $H_1 \subseteq V$.

A map $s : A \hookrightarrow X$ is a *SSDR-map* iff $s$ embeds $A$ as a shape strong deformation retract of $X$.

**Proposition 1.2.** *Let $A$ is a closed subset of of $X$. Then $X \times 0 \cup A \times I \hookrightarrow X \times I$ is a SSDR-map. If $A \hookrightarrow X$ is an SSDR-map, then $X \times 0 \cup A \times I \cup X \times 1 \hookrightarrow X \times I$ is also an SSDR-map.*

A closed subset $A$ of a space $X$ is *strong infinite deformation retract* of $X$ iff there exists a *strong infinite deformation of $X$ onto $A$*, i.e. a map

$$D : X \times [0, 1) \to X$$

such that:

(i) $D(a, t) = a$, if $a \in A$, $t \in [0, 1)$;

(ii) for any neighborhood $U$ of $A$ in $X$ there exists $\lambda \in [0, 1)$ such that

$$D(X \times [\lambda, 1)) \subset U.$$

Note that if $A$ is a strong infinite deformation retract of $X$, then $A \hookrightarrow X$ is an SSDR-map.

A space $Y$ is a *fibrant* space iff it has the extension property with respect to SSDR-maps in the following sense: for every SSDR-map $s : A \hookrightarrow X$ and every map $A \to Y$ there exists a map $\bar{f} : X \to Y$ such that $\bar{f} \circ s = f$.

Every ANR is a fibrant space. Inverse limits of inverse sequences, consisting of fibrants and Hurewicz fibrations as bonding maps, are fibrant. Compact metric topological groups are fibrant. Retracts of fibrant spaces are obviously fibrant.

A *fibrant extension* of $X$ consists of a fibrant space $\tilde{X}$ and an SSDR-map $s : X \hookrightarrow \tilde{X}$. Throughout the paper, dealing with fibrant extensions $\tilde{X}$ of $X$, we always suppose that $X \subseteq \tilde{X}$.

The crucial point is the following statement (see [Ca2], [Ca3]):

**Proposition 1.3.** *Every compact metric space has a fibrant extension.*

Each of ANR-resolutions $\underline{q} : X \to \underline{X}$ (more generally, resolutions consisting of fibrant spaces) of a compact space $X$ can be used to construct its fibrant extension. Namely, so called *cotelescope* $\tilde{X} = coTel(\underline{X})$ of $X$ (see [L]) with the natural embedding $s : X \hookrightarrow \tilde{X}$ serves as a fibrant extension for $X$. Moreover, $s(X)$ is a strong infinite deformation retract of $\tilde{X}$.

The cotelescope construction can be realized in the category $Map\mathcal{M}$. In this category Hurewicz fibrations of fibrant spaces can be regarded as "fibrant" objects. For details, see [BZ]. In a natural way the concepts of strong deformation and strong infinite deformation are defined in $Map\mathcal{M}$.

For example, *a strong infinite deformation of* $\tilde{p}$ *onto* $p$ , in the category $Map\mathcal{M}$, can be represented by a pair $(D, H)$ of strong infinite deformations

$$D : \tilde{E} \times [0,1) \to \tilde{E}\ ,\ H : \tilde{B} \times [0,1) \to \tilde{B}$$

onto $E$ and $B$ respectively, connected with the relation

$$\tilde{p} \circ D = H \circ (\tilde{p} \times id_{[0,1)}).$$

As a consequence of Theorem 1.1 and the cotelescope construction we get the following

**Theorem 1.4.** ([BZ],Th.9, p.207) *If* $p : E \to B$ *is a shape fibration of metric compacta* $E$ *and* $B$*, then there exists a Hurewicz fibration* $\tilde{p} : \tilde{E} \to \tilde{B}$ *of fibrant extensions* $\tilde{E}$ *and* $\tilde{B}$ *of* $E$ *and* $B$ *respectively such that the following diagram commutes*

$$\begin{array}{ccc} E & \xrightarrow{\ i\ } & \tilde{E} \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle \tilde{p}} \\ B & \xrightarrow{\ j\ } & \tilde{B} \end{array}$$

*where $i : E \hookrightarrow \tilde{E}$ and $j : B \hookrightarrow \tilde{B}$ are SSDR-maps.*

*Moreover, there exists a strong infinite deformation of $\tilde{p}$ onto $p$ in the category $Map\mathcal{M}$.*

If $i : X \hookrightarrow \tilde{X}$ and $j : Y \hookrightarrow \tilde{Y}$ are fibrant extensions of compact metric spaces $X$ and $Y$, then a *strong shape morphism* $X \to Y$ is a homotopy class $[\tilde{f}]$ of a map $\tilde{X} \to \tilde{Y}$. For each map $f : X \to Y$, there is an extension $\tilde{f} : \tilde{X} \to \tilde{Y}$, $\tilde{f} \circ i = j \circ f$, and it is "homotopicly unique" in the following sense: if $\tilde{f}' : \tilde{X} \to \tilde{Y}$ is other extension of $f$, then $\tilde{f} \simeq \tilde{f}'$ $rel.X$. Moreover, these extensions of homotopic maps are homotopic. All this permits to define the canonical functor $h\mathcal{CM} \to ssh\mathcal{CM}$, as well as the strong shape category $ssh\mathcal{CM}$ itself. For the whole story, see [Ca2].

Note now that the map $\tilde{p}$ in Theorem 1.4 can be used to study strong shape properties of $p$. So $p$ is a ssh-domination iff there exists a map $g : \tilde{B} \to \tilde{E}$ such that $\tilde{p} \circ g \simeq id_{\tilde{B}}$. In fact, it follows that $\tilde{p} \circ \tilde{s} = id_{\tilde{B}}$ for some map $\tilde{s} : \tilde{B} \to \tilde{E}$, because $\tilde{p}$ is a fibration.

In Section 3 we use the following

**Proposition 1.5.** ([Ca1]) *Suppose $\tilde{X}$ is a fibrant extension of connected compactum $X$. $X$ is an FANR if and only if $\tilde{X}$ is an ANR.*

**Corollary 1.6.** *If connected FANR is a fibrant space, then it is ANR.*

## 2. Preserving a fibrant space

Note that in order to verify that a compact space $B$ is fibrant, it is sufficient to check the extension property given in the definition with respect only to those SDR-maps $s : A \to X$ in which $A$ is compact. Indeed, applying the property to a fibrant extension $s : B \to \tilde{B}$ we extend $id : B \to B$ to a retraction $\tilde{B} \to B$.

**Theorem 2.1.** *Let $p : E \to B$ be a shape fibration and a ssh-domination of metric compacta $E$ and $B$. If $E$ is a fibrant space, then $B$ is also a fibrant space.*

**Proof:** Let $A$ be a compact metric space and $j : A \hookrightarrow X$ be an SSDR-inclusion. For every map $f : A \to B$ we shall construct an extension $F : X \to B$.

Applying Theorem 1.4 to the map $p : E \to B$ we can note that the map $\tilde{p} : \tilde{E} \to \tilde{B}$ between fibrant extensions has a homotopy right inverse, because $p$ is an ssh-domination. Since $\tilde{p}$ is simultaneously a Hurewicz fibration, it follows that $\tilde{p}$ admits a section $\tilde{s} : \tilde{B} \to \tilde{E}$, $\tilde{p} \circ \tilde{s} = id_{\tilde{B}}$. The map $\overline{f} = \tilde{s} \mid_B \circ f : A \to \tilde{E}$ has an extension $\overline{F} : X \to \tilde{E}$, because $\tilde{E}$ is a fibrant space. Let $(D, H)$ be a strong infinite deformation of $\tilde{p}$ onto $p$. We define $\overline{G} : X \times [0,1) \to B$ by $\overline{G}(x,t) = (p \circ r \circ D)(\overline{F}(x), t)$, where $r : \tilde{E} \to E$ is a retraction. This retraction exists because $E$ is a fibrant space and $i : E \hookrightarrow \tilde{E}$ is an SSDR-map.

Note that for every open cover $\mathcal{U}$ of $E$ in $\tilde{E}$ there exists $\lambda \in [0,1)$ such that for any $t \in [\lambda, 1)$ and $e \in \tilde{E}$ we get $r(D(e,t)), D(e,t) \in U$ for some $U \in \mathcal{U}$. Consequently, every open cover $\mathcal{V}$ of $B$ in $\tilde{B}$ admits $\lambda \in [0,1)$ satisfying the condition: for any $t \in [\lambda, 1)$ and $e \in \tilde{E}$ we have

$$(p \circ r \circ D)(e,t), (p \circ D)(e,t) \in V$$

for some $V \in \mathcal{V}$. Particularly, if $e = \overline{f}(a)$ for $a \in A$, it follows that $\overline{G}(a,t), f(a) \in V$, because

$$\begin{aligned}(p \circ D)(\overline{f}(a), t) &= (p \circ D)(\tilde{s}(f(a)), t) = H(\tilde{p}(\tilde{s}(f(a))), t) \\ &= H(f(a), t) = f(a).\end{aligned}$$

Therefore, we can extend the restriction $\overline{G} \mid_{A \times [0,1)}$ to the homotopy
$G : A \times [0,1] \to B$, defining $G(a,1) = f(a)$ for $a \in A$.

Since $A$ is compact, for a sequence of numbers $\varepsilon_0 = diamB > \varepsilon_1 > ... > \varepsilon_n > ...$ tending to 0, we can choose a sequence of numbers $t_0 = 0 < t_1 < ... < t_n < ...$ tending to 1 such that for all $a \in A$ and every $n$ the relation $diamG(\{a\} \times [t_n, 1]) < \varepsilon_n$ takes place. After that we can find a sequence $U_0 = X, U_1, ...U_n, ...$ of open neighborhoods of $A$ in $X$, satisfying the following conditions:

$$\bigcap U_n = A;\ \overline{U}_{n+1} \subset U_n \text{ and } diam\overline{G}(\{x\} \times [t_k, t_{n+1}]) < \varepsilon_k$$

for $k \leq n$ and all $x \in U_n$.

Choosing a family of open subsets $\{V_i, i = 1, 2, ..\}$ for which $V_1 = X$ and $U_n \supset \overline{V}_{n+1} \supset V_{n+1} \supset \overline{U}_{n+1}$ one can find a sequence of continuous functions

$$\varphi_n : \overline{V}_n \setminus U_{n+1} \longrightarrow [t_{n-1}, t_n]$$

such that $\varphi_n^{-1}(t_{n-1}) = \overline{V}_n \setminus U_n$ and $\varphi_n^{-1}(t_n) = \overline{V}_{n+1} \setminus U_{n+1}$. This sequence determines a continuous function $\varphi : X \setminus A \to [0, 1)$.

It is not hard to verify, that the map $F : X \to B$, given by $F(x) = \overline{G}(x, \varphi(x))$ for $x \in X \setminus A$ and $F(x) = f(x)$ for $x \in A$, is a continuous extension of $f$.

## 3. Applications

The following lemma concerns ordinary shape domination (for a definition of shape domination, see [MS82], p.27):

**Lemma 3.1.** *Let $p : E \to B$ be a shape fibration and a shape domination of metric compact spaces. If $p \to \tilde{p}$, $\tilde{p} : \tilde{E} \to \tilde{B}$, is a fibrant extension of p as in Theorem 1.4 , then for each open cover $\mathcal{U}$ of $B$ in $\tilde{B}$ there exists a map $g : \tilde{B} \to \tilde{E}$ such that $\tilde{p} \circ g \mid_B$ and $i : B \hookrightarrow \tilde{B}$ are $\mathcal{U}$-near.*

**Proof:** We shall need more information about $\tilde{p}$ involved in Theorem 1.4. Therefore we partially recall how $\tilde{p}$ is constructed in [BZ]. For ANR-resolution $p \to \underline{p}$, $\underline{p} = \{p_i : E_i \to B_i\}$, we obtain the following commutative diagram:

$$\begin{array}{ccccccccc}
\tilde{p}_1 & \longleftarrow & \cdots & \tilde{p}_i & \xleftarrow{\tilde{\alpha}_i^{i+1}} & \tilde{p}_{i+1} & \longleftarrow & \cdots & \tilde{p} \\
\uparrow \sigma_1 & & & \uparrow \sigma_i & & \uparrow \sigma_{i+1} & & & \uparrow \sigma \\
p_1 & \longleftarrow & \cdots & p_i & \xleftarrow{\alpha_i^{i+1}} & p_{i+1} & \longleftarrow & \cdots & p
\end{array}$$

In this diagram $\tilde{p} = \varprojlim\{\tilde{p}_i, \tilde{\alpha}_i^j\}$, every morphism $\sigma_i = (t_i, s_i)$, where $t_i : E_i \hookrightarrow \tilde{E}_i, s_i : B_i \hookrightarrow \tilde{B}_i$, embeds $p_i$ in $\tilde{p}_i$ as strong deformation retract (in $Map\mathcal{M}$). Actually, we may consider $E_i$ and $B_i$ as subsets of $\tilde{E}_i$ and $\tilde{B}_i$ respectively. All $p_i$ and $\tilde{p}_i$ are fibrations of ANRs. Moreover, each morphism $\tilde{\alpha} = (\tilde{q}_i^{i+1}, \tilde{r}_i^{i+1})$ is a pair of fibrations (in fact, it satisfies some more delicate property, see [BZ] and also [EH] §3.2, but we do not use it here ).

Let $\mathcal{U}$ be an open cover of $B$ in $\tilde{B}$. Since $B$ is compact, we can find an index $n$ and an open cover $\mathcal{V}$ of $\tilde{r}_n(B)$ in $\tilde{B}_n$ such that $\{\tilde{r}_n^{-1}(V) : V \in \mathcal{V}\}$ is a refinement of $\mathcal{U}$. In particular, for each $x \in \tilde{r}_n(B)$ we have $\tilde{r}_n^{-1}(x) \subseteq U$ for some $U \in \mathcal{U}$.

The maps $t_i$, $s_i$ are homotopy equivalences, because they are SDR-maps. Therefore we can use $\underline{\tilde{p}}$ (instead of $\underline{p}$) to study shape properties of $p$.

Suppose now that $p$ is a shape domination. W.l.o.g. we may assume that there exist maps $f_i : \tilde{B}_{i+1} \to \tilde{E}_i$, $i \in \mathbf{N}$ which satisfy the conditions:

$$(a)\ \tilde{q}_i^{i+1} \circ f_{i+1} \simeq f_i \circ \tilde{r}_{i+1}^{i+2}, \quad (b)\ \tilde{p}_i \circ f_i \simeq \tilde{r}_i^{i+1}.$$

Since $\tilde{p}_n$ is a fibration, we find a map $g_n : \tilde{B}_{n+1} \to \tilde{E}_n$, using $(b)$ for $i = n$, such that

$$g_n \simeq f_n, \quad \tilde{p}_n \circ g_n = \tilde{r}_n^{n+1}.$$

According to $(a)$, $\tilde{q}_n^{n+1} \circ f_{n+1} \simeq f_n \circ \tilde{r}_{n+1}^{n+2} \simeq g_n \circ \tilde{r}_{n+1}^{n+2}$. Now since $\tilde{q}_n^{n+1}$ is a fibration, we find a map $g_{n+1} : \tilde{B}_{n+2} \to \tilde{E}_{n+1}$ with $g_{n+1} \simeq f_{n+1}$ and $\tilde{q}_n^{n+1} \circ g_{n+1} = g_n \circ \tilde{r}_{n+1}^{n+2}$.

Similarly, for each $i \geq n$ we get a map $g_i : \tilde{B}_{i+1} \to \tilde{E}_i$ with

$$g_i \simeq f_i, \quad \tilde{q}_i^{i+1} \circ g_{i+1} = g_i \circ \tilde{r}_{i+1}^{i+2}.$$

Hence the maps $g_i$ induce a map $g : \tilde{B} \to \tilde{E}$ such that $\tilde{q}_i \circ g = g_i \circ \tilde{r}_{i+1}$ for each $i \geq n$. This is a required map. Indeed, $\tilde{r}_n \circ \tilde{p} \circ g = \tilde{p}_n \circ \tilde{q}_n \circ g = \tilde{p}_n \circ g_n \circ \tilde{r}_{n+1} = \tilde{r}_n$, and so for each $b \in B$ we have $b, \tilde{p} \circ g(b) \in \tilde{r}_n^{-1}(\tilde{r}_n(b))$. By the choice of $n$ there is $U \in \mathcal{U}$ such that $b, \tilde{p} \circ g(b) \in U$. In other words, $\tilde{p} \circ g \mid_B$ and $i : B \hookrightarrow \tilde{B}$ are $\mathcal{U}$-near.

**Proposition 3.2.** *Let $p : E \to B$ be a shape fibration and a shape domination of metric compact spaces. If $B$ is a connected FANR, then $p$ is ssh-domination.*

**Proof:** Let $p \to \tilde{p}$, $\tilde{p} : \tilde{E} \to \tilde{B}$, be a fibrant extension of $p$ as in Theorem 1.4. Since $B$ is a connected FANR, $\tilde{B}$ is ANR by Proposition 1.5. Hence there exists an open cover $\mathcal{U}$ of $\tilde{B}$ such that $\mathcal{U}$-near maps in $\tilde{B}$ are homotopic (see [MS82], p.39). Applying Lemma to this cover we get a map $g : \tilde{B} \to \tilde{E}$ and a homotopy $H : B \times I \to \tilde{B}$ with $H_0 = \tilde{p} \circ g \mid_B$ and $H_1 = i$. The Proposition will be proved if we show that $H$ can be extended to a homotopy $\tilde{p} \circ g \simeq id_{\tilde{B}}$. But it easily follows from Proposition 1.2: the map $G : \tilde{B} \times 0 \cup B \times I \cup \tilde{B} \times 1 \to \tilde{B}$ given by $G(\tilde{b}, 0) = \tilde{p} \circ g(\tilde{b})$, $G(b,t) = H(b,t)$, $G(\tilde{b}, 1) = \tilde{b}$, $\tilde{b} \in \tilde{B}, b \in B$ has an extension $\bar{H} : \tilde{B} \times I \to \tilde{B}$, because $i : B \hookrightarrow \tilde{B}$ is an SSDR-map and $\tilde{B}$ is fibrant.

**Corollary 3.3.** (see [CMT]) *Let a map $p : E \to B$ be a shape fibration and shape domination of connected compact metric spaces. If $E$ is an ANR, then $B$ is also an ANR.*

**Proof:** Since $p$ is a shape domination, $B$ is FANR. By the Proposition $p$ is an ssh-domination, and we can apply Theorem 2.1. Hence $B$ is fibrant, because $E$ is fibrant, and so $B$ is a retract of $\tilde{B}$. But $\tilde{B}$ is ANR, and so is $B$.

Now let us suppose that a compact metric space $E$ is an *approximate ANR-space* (*AANR*) in sense of Clapp or, in other words, $E$ is an approximate polyhedron in the sense of S. Mardešić [M]. If there is an infinite deformation of some metric space $\tilde{E}$ onto E, then it is not hard to verify that $E$ is an $\varepsilon$-retract of $\tilde{E}$ for every $\varepsilon > 0$.

Applying Theorem 1.4 to the shape fibration and ssh-domination $p : E \to B$, where $E$ is an approximate polyhedron, we can state the following:
for every $\varepsilon > 0$ there exists a $\delta$-retraction $r : \tilde{E} \to E$ for some $\delta > 0$ and a section $s : B \to \tilde{E}$ such that $\rho(prs(b), b) < \varepsilon$ for all $b \in B$ (where $\rho$ is a metric on $B$). This means that $B$ is approximately dominated by $E$. Hence we get the following result.

**Theorem 3.4.** *If a map $p : E \to B$ of compact metric spaces is a shape fibration and ssh-domination and, moreover, if $E$ is an approximate polyhedron, then $B$ is also an approximate polyhedron.*

Note that compact $AANR$-spaces in the sense of Noguchi coincide with those $FANR$-spaces, which are $AANR$-spaces in the sense of Clapp (see [K]). Hence we immediately obtain the following

**Corollary 3.5.** *If a map $p : E \to B$ of connected compact metric spaces is a shape fibration and a shape domination and, moreover, $E$ is an $AANR$ in the sense of Noguchi, then $B$ is also an $AANR$ in the sense of Noguchi.*

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Benemérita Universidad Autónoma de Puebla, Av. San Claudio y Rio Verde, Ciudad Universitaria, Colonia San Manuel, CP 72570, Puebla, Pue., México