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CONTRIBUTED PROBLEMS

The Problems Editor invites anyone who has published a paper in *Topology Proceedings* or has attended a Spring or Summer Topology Conference to submit problems to this section. They need not be related to any articles which have appeared in *Topology Proceedings* or elsewhere, but if they are, please provide full references. Please define any terms not in a general topology text nor in referenced articles.

Problems which are stated in, or relevant to, a paper in this volume are accompanied by the title of the paper where further information about the problem may be found. Comments of the proposer or submitter of the questions are so noted; comments of the Problems Editor are not specially noted. Information on the status of previously posed questions is always welcome. Submission of questions and comments by email in TEX form is strongly encouraged, either to topolog@mail.auburn.edu or directly to the Problems Editor at mayer@math.uab.edu.

F. Continuum Theory.

46. (Carl Seaquist, *"A continuous decomposition of the plane into acyclic continua each of which contains an arc"*) Does there exist a continuous decomposition G of the plane into acyclic continua so that for every point x, there is an arc A and an element $g \in G$ such that $x \in A \subset g$?

G. Mappings of Continua and Euclidean Spaces.

40. (Francis Jordan, *"Almost continuous images of* $\mathbb{R}$ *and* ∞*-ods"*) Characterize the continua which are the almost continuous images of the reals.

L. Topological Algebra.

46–48. Comments of the proposer. In *Handbook of Boolean Algebras*, page 540, the following result is proved: *If MA(k) holds, and A is a Boolean algebra with infinitely many atoms such that $|A| = k$, then S^∞ can be isomorphically embedded in Aut(A).*

46. (Alfino Gianlotta, *"Combinatorial and topological aspects of measure-preserving functions"*) Are S^∞ and A comparable as groups? That is, does there exist an emdedding of one of them into the other one?

The above question is probably very difficult, yet the following weaker version of it seems to be very interesting as well.

47. (Alfino Gianlotta) Are S^∞ and A comparable as subgroups of Aut(Pow$\mathbb{N}$/Fin)?

This question makes sense, since both groups can be isomorphically embedded into Pow$\mathbb{N}$/Fin, as is proved in the paper (in progress) "Embeddings into Pow$\mathbb{N}$/Fin and extensions of automorphisms," by A. Bella, A. Dow, and P. Ursino.

48. (Alfino Gianlotta) Is there a formula that gives the order of a (particular) element γ in $(S, \circ)$ in terms of the parameters of the shifts of which γ is the composition?

A partial answer for the composition of two rational shifts has been found by the authors.

P. Products, Hyperspaces, Remainders, and Similar Constructions.

50–52. Comments of the proposer. Given a continuum X, consider a class $\mathcal{F}_X$ of continua Y such that

(1) no member of $\mathcal{F}_X$ is homeomorphic to X;
(2) no two distinct members of $\mathcal{F}_X$ are homeomorphic;
(3) the hyperspaces $C(X)$ and $C(Y)$ are homeomorphic, for each $Y \in \mathcal{F}_X$;
(4) if Z is a continuum such that the hyperspaces $C(Z)$ and $C(X)$ are homeomorphic, then either Z is homeomorphic to X or Z is homeomorphic to some member Y of $\mathcal{F}_X$.

A continuum X is said to have *unique hyperspace* iff the class $\mathcal{F}_X$ is empty. If the class $\mathcal{F}_X$ is nonempty and finite, we say that X has *almost unique hyperspace*.

50. (Gerardo Acosta, *"On compactifications of the real line and unique hyperspace"*) Let X be a fan without the property of Kelley. Is it true that X does not have almost unique hyperspace?

51. (Gerardo Acosta) Let X be an indecomposable continuum such that each proper and nondegenerate subcontinuum of X is a finite graph. Does X have unique hyperspace?

52. (Gerardo Acosta) For a metric compactification of the space $V = (-\infty, \infty)$ and connected and nondegenerate remainder R, we write $X = V \cup R$ and define

$$R_1 = \bigcap_{n \in \mathbb{N}} \mathrm{Cl}_X((n, \infty)) \quad \text{and} \quad R_2 = \bigcap_{n \in \mathbb{N}} \mathrm{Cl}_X((-\infty, -n)).$$

Let us assume that $R_1 \neq R_2$. Is there a continuum Y, not homeomorphic to X, such that the hyperspaces $C(X)$ and $C(Y)$ are homeomorphic? What is the cardinality of the class $\mathcal{F}_X$?