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TOPOLOGIES ON FUNCTION SPACES AND THE COORDINATE CONTINUITY

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Abstract

We obtain variations of the splitting and jointly continuous topologies replacing in its definitions the continuity of mappings (of the product spaces) by the coordinately continuity. These variation are used to study the pointwise topology and its relations to some other well known topologies on function spaces.

1. Introduction

We denote by Y and Z two fixed topological spaces and by $C(Y,Z)$ the set of all continuous maps of Y into Z. If τ is a topology on the set $C(Y,Z)$, then the corresponding topological space is denoted by $C_\tau(Y,Z)$.

Let X be a topological space and F a map of $X \times Y$ into Z. We denote by F_x the map of Y into Z for which $F_x(y) = F(x,y)$ for every $y \in Y$ and, by F^y the map of X into Z for which $F^y(x) = F(x,y)$ for every $x \in X$. A mapping $F : X \times Y \to Z$ is said to be *coordinately continuous* if the maps F_x and F^y are continuous for every $x \in X$ and $y \in Y$. It is clear that the continuity of F implies the coordinate continuity of this map. Suppose that $F : X \times Y \to Z$ is a coordinately continuous map. By $\widehat{F}$ we denote the map of X into the set $C(Y,Z)$, for

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which $\widehat{F}(x) = F_x$, for every $x \in X$. Let G be a map of X into $C(Y,Z)$. By $\widetilde{G}$ we denote the map of $X \times Y$ into Z, for which $\widetilde{G}(x,y) = G(x)(y)$, for every $(x,y) \in X \times Y$.

By $\mathcal{A}$ we denote an arbitrary fixed family of topological spaces.

We start given some well known notions concerning the topologies on the set $C(Y,Z)$.

A topology τ on $C(Y,Z)$ is called *splitting* if for every space X, the continuity of a map $F : X \times Y \to Z$ implies that of the map $\widehat{F} : X \to C_\tau(Y,Z)$. A topology τ on $C(Y,Z)$ is called *jointly continuous* if for every space X, the continuity of a map $G : X \to C_\tau(Y,Z)$ implies that of the map $\widetilde{G} : X \times Y \to Z$ (see, [1], [2], [3], [4] and [11]).

If in the above definitions it is assumed that the space X belongs to $\mathcal{A}$, then the topology τ is called $\mathcal{A}-$*splitting* (respectively, $\mathcal{A}-$*jointly continuous*) (see [5] and [7]). Also, for some other variations of splitting and jointly continuous topologies see [8].

Let $\mathcal{O}(Y)$ be the family of all open sets of the space Y. The *Scott topology* on $\mathcal{O}(Y)$ (see, for example, [6]) is defined as follows: a subset $I\!H$ of $\mathcal{O}(Y)$ is open if:

(α) the conditions $U \in I\!H$, $V \in \mathcal{O}(Y)$, and $U \subseteq V$ imply $V \in I\!H$, and

(β) for every collection of open sets of Y, whose union belongs to $I\!H$, there are finitely many elements of this collection whose union also belongs to $I\!H$.

The *strong Scott topology* on $\mathcal{O}(Y)$ (see, [10]) is defined as follows: a subset $I\!H$ of $\mathcal{O}(Y)$ is open if:

(α) the conditions $U \in I\!H$, $V \in \mathcal{O}(Y)$, and $U \subseteq V$ imply $V \in I\!H$, and

(β) for every open cover of Y, there are finitely many elements of this cover whose union belongs to $I\!H$.

The *pointwise topology* (see, for example, [3] and [11]) τ_p on $C(Y,Z)$ is the topology for which the family of all sets of the form

$$(\{y\}, U) = \{f \in C(Y, Z) : f(y) \in U\},$$

where $y \in Y$ and $U \in \mathcal{O}(Z)$, is a subbasis.

The *compact open* (see, [4]) topology τ_c on $C(Y, Z)$ is the topology for which the family of all sets of the form

$$(K, U) = \{f \in C(Y, Z) : f(K) \subseteq U\},$$

where K is a compact subset of Y and $U \in \mathcal{O}(Z)$, is a subbasis.

The *Isbell topology* τ_{is} (respectively, *strong Isbell topology* τ_{s-is}) on $C(Y, Z)$ (see, [11] and [10]) is the topology for which the family of all sets of the form

$$(I\!H, U) = \{f \in C(Y, Z) : f^{-1}(U) \in I\!H\},$$

where $I\!H$ is Scott (respectively, strong Scott) open in $\mathcal{O}(Y)$ and $U \in \mathcal{O}(Z)$, is a subbasis.

The *bounded open* topology τ_{bo} on $C(Y, Z)$ (see, [9]) is the topology for which the family of all sets of the form:

$$(K, U) = \{f \in C(Y, Z) : f(K) \subseteq U\},$$

where K is a bounded set of Y (a subset K of Y is said to be *bounded* if every open cover of Y has a finite subcover for K) and $U \in \mathcal{O}(Z)$, is a subbasis.

The *open open* topology (see, [13]) τ_{oo} on $C(Y, Z)$ is the topology for which the family of all sets of the form:

$$(V, U) = \{f \in C(Y, Z) : f(K) \subseteq U\},$$

where $V \in \mathcal{O}(Y)$ and $U \in \mathcal{O}(Z)$, is a subbasis.

The map $e : C(Y, Z) \times Y \to Z$, which is defined by $e(f, y) = f(y)$ for every $f \in C(Y, Z)$ and $y \in Y$ is called *the evaluation map* (see, [1]).

A space X is called *corecompact* (see, for example, [10]) if for every $x \in X$ and for every open neighbourhood U of x, there exists an open neighbourhood V of x such that the subset V is bounded in U.

Below, we give some well known results:

(1) The pontwise topology, the compact open topology and the Isbell topology on $C(Y, Z)$ are always splitting (see, for example, [1], [2], [3], [4] and [11]).

(2) The compact open topology on $C(Y, Z)$ is jointly continuous if Y is localy compact (a space Y is *localy compact* if for every $y \in Y$ there exists a neighbourhood U of y such that Cl(U) is a compact subspace of X). In this case the compact open topology is also the greatest splitting topology (see, [4] and [2]).

(3) The Isbell topology on $C(Y, Z)$ is jointly continuous if Y is a corecompact space. In this case the Isbell topology is also the greatest splitting topology (see, for example, [10] and [11]).

(4) The Isbell topology on $C(Y, \mathbf{S})$, where $\mathbf{S}$ is the Sierpinski space (that is $\mathbf{S} = \{0, 1\}$ with the topology $\tau = \{\emptyset, \{0\}, \{0, 1\}\}$) is the greatest splitting topology (see, [12]).

(5) The strong Isbell topology on $C(Y, Z)$ is jointly continuous if Y is locally bounded (see, [10]).

(6) The open open topology on $C(Y, Z)$ is always jointly continuous (see, [13]).

2. Coordinately $\mathcal{A}$-splitting and $\mathcal{A}$-jointly Continuous Topologies

Definition 2.1. A topology τ on $C(Y, Z)$ is called *coordinately splitting* if for every space X the coordinate continuity of a map $F : X \times Y \to Z$ implies the continuity of the map $\widehat{F} : X \to C_\tau(Y, Z)$.

A topology τ on $C(Y, Z)$ is called *coordinately jointly continuous* if for every space X the continuity of a map $G : X \to C_\tau(Y, Z)$ implies the coordinate continuity of the map $\widetilde{G} : X \times Y \to Z$.

If in the above definitions the space X is assumed to be an element of $\mathcal{A}$, then τ is called *coordinately $\mathcal{A}-$splitting* (respectively, *coordinately $\mathcal{A}-$jointly continuous*).

By definitions it follows immediately that if $\mathcal{A}_1$ and $\mathcal{A}_2$ two families of spaces such that $\mathcal{A}_1 \subseteq \mathcal{A}_2$, then any coordinately $\mathcal{A}_2-$splitting (respectively, coordinately $\mathcal{A}_2-$jointly continuous) topology is also coordinately $\mathcal{A}_1-$splitting (respectively, coordinately $\mathcal{A}_1-$jointly continuous).

Theorem 2.1. *The following propositions are true:*

(1) Every coordinately $\mathcal{A}-$spitting topology on $C(Y,Z)$ is $\mathcal{A}-$splitting topology.

(2) Every $\mathcal{A}-$jointly continuous topology on $C(Y,Z)$ is coordinately $\mathcal{A}-$jointly continuous topology.

The proof of this theorem is clear.

Below, we give some results concerning coordinately splitting and coordinately jointly continuous topologies on $C(Y,Z)$. These results are similar to the results of Section I of [5] concerning splitting and jointly continuous topologies on $C(Y,Z)$. The corresponding proofs are also similar and so are omitted.

Theorem 2.2. *Let τ be a topology on $C(Y,Z)$ such that $C_\tau(Y,Z) \in \mathcal{A}$. Then the following propositions are true:*

(1) The topology τ is coordinately $\mathcal{A}-$jointly continuous if and only if the evaluation map $e : C_\tau(Y,Z) \times Y \to Z$ is coordinately continuous.

(2) If τ is larger than a coordinately $\mathcal{A}-$jointly continuous topology, then τ is also coordinately $\mathcal{A}-$jointly continuous.

(3) If τ is coordinately $\mathcal{A}-$jointly continuous, then τ is larger than any coordinately $\mathcal{A}-$splitting topology.

(4) If τ is coordinately $\mathcal{A}-$jointly continuous, then τ is coordinately jointly continuous topology.

Corollary 2.1. *The following propositions are true:*

(1) A topology τ *on* $C(Y, Z)$ *is coordinately jointly continuous if and only if the evaluation map* $e : C_\tau(Y, Z) \times Y \to Z$ *is coordinately continuous.*

(2) A topology on $C(Y, Z)$ *larger than a coordinately jointly continuous topology is also coordinately jointly continuous.*

(3) Any coordinately jointly continuous topology on $C(Y, Z)$ *is larger than any coordinately splitting topology.*

Theorem 2.3. *The following propositions are true:*

(1) A topology on $C(Y, Z)$ *smaller than a coordinately* $\mathcal{A}-$*splitting topology is also coordinately* $\mathcal{A}-$*splitting.*

(2) On the set $C(Y, Z)$ *there exists the greatest coordinately* $\mathcal{A}-$*splitting topology, which is denoted by* $\tau(\mathcal{A})$.

(3) Let $\mathcal{A}_i$, $i \in I$, *be a family of spaces and* $\mathcal{A} = \bigcup\{\mathcal{A}_i : i \in I\}$. *Then,* $\tau(\mathcal{A}) = \bigcap\{\tau(\mathcal{A}_i) : i \in I\}$.

(4) Let $\mathcal{A}_i$, $i \in I$, *be a family of spaces and* $\mathcal{A} = \bigcap\{\mathcal{A}_i : i \in I\} \neq \emptyset$. *Then* $\vee\{\tau(\mathcal{A}_i) : i \in I\} \subseteq \tau(\mathcal{A})$.

(5) $\tau(\mathcal{A}) = \bigcap\{\tau(\{X\}) : X \in \mathcal{A}\}$.

Corollary 2.2. *The following propositions are true:*

(1) A topology on $C(Y, Z)$ *smaller than a coordinately splitting topology is also coordinately splitting.*

(2) If τ *is a coordinately splitting and coordinately jointly continuous topology, then* τ *is uniquely defined.*

Definition 2.2. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be two classes of spaces. We say that these families are *coordinately equivalent* and write $\mathcal{A}_1 \overset{c-\tau}{\sim} \mathcal{A}_2$ if and only if: (α) a topology τ on $C(Y, Z)$ is an coordinately $\mathcal{A}_1-$splitting if and only if τ is coordinately $\mathcal{A}_2-$splitting and (β) a topology τ on $C(Y, Z)$ is coordinately $\mathcal{A}_1-$jointly continuous topology if and only if τ is coordinately $\mathcal{A}_2-$jointly continuous.

Theorem 2.4. *There exists a space* $X(\mathcal{A})$ *such that*

$$\mathcal{A} \overset{c-\tau}{\sim} \{X(\mathcal{A})\}.$$

Corollary 2.3. *There exists a space* X *such that: (*α*) a topology on* $C(Y,Z)$ *is coordinately splitting if and only if this topology is coordinately* $\{X\}-$*splitting and (*β*) a topology on* $C(Y,Z)$ *is coordinately jointly continuous if and only if this topology is coordinately* $\{X\}-$*jointly continuous.*

Notations 2.1. For every space X we denote by $S(X \times Y, Z)$ the set of all coordinately continuous maps of the space $X \times Y$ into the space Z. We note that the set $S(X \times Y, Z)$ contains the set $C(X \times Y, Z)$.

Definition 2.3. Let τ be a coordinately splitting topology on $C(Y,Z)$. The *coordinately exponential function*

$$E^{\tau} : S(X \times Y, Z) \to C(X, C_{\tau}(Y,Z))$$

is defined setting $E^{\tau}(F) = \widehat{F}$, for every $F \in S(X \times Y, Z)$.

We note that this function is well defined since τ is coordinately splitting. Also, the restriction of E^{τ} on $C(X \times Y, Z)$ coincides with the exponential function E (see, for example, [11]).

It is easy to verify the following theorem:

Theorem 2.5. *If for every space* X *the mapping* E^{τ} *is onto, then* τ *is a coordinately jointly continuous topology.*

3. On the Pointwise Topology

Theorem 3.1. *The pointwise topology* τ_p *on* $C(Y,Z)$ *is coordinately* $\mathcal{A}-$*jointly continuous.*

Proof. Let $G : X \to C_{\tau_p}(Y,Z)$ be a continuous map, where X is an element of $\mathcal{A}$. We need to prove that the map $\widetilde{G} : X \times Y \to Z$ is coordinately continuous.

Let x be a fixed element of X. Then, the map $\widetilde{G}_x : Y \to Z$ (for which $\widetilde{G}_x(y) = \widetilde{G}(x,y) = G(x)(y)$ for every $y \in Y$) coincides with $G(x)$, and therefore, it is continuous.

Now, let y be a fixed element of Y. Consider the map $\widetilde{G}^y : X \to Z$ (for which $\widetilde{G}^y(x) = \widetilde{G}(x,y)$ for every $x \in X$) and prove that this map is continuous. Indeed, let x be an element of X and W an open neighbourhood of $\widetilde{G}^y(x)$ in Z, that is, $\widetilde{G}(x,y) = G(x)(y) \in W$. Then, $y \in (G(x))^{-1}(W)$. By definition of the pointwise topology the subset $(\{y\}, W)$ of $C_{\tau_p}(Y,Z)$ is open and $G(x) \in (\{y\}, W)$. Since G is continuous there exists an open neighbourhood V_x of x in X such that:

$$G(V_x) \subseteq (\{y\}, W).$$

By the definition of $\widetilde{G}$, $\widetilde{G}(V_x \times \{y\}) \subseteq W$ and, therefore, $\widetilde{G}^y(V_x) \subseteq W$. Hence, the map $\widetilde{G}^y$ is continuous.

Thus, the map $\widetilde{G} : X \times Y \to Z$ is coordinately continuous which means that the topology τ_p is coordinately $\mathcal{A}-$jointly continuous. □

Corollary 3.1. *Let τ be one of the following topologies: the compact open topology, the Isbell topology, the strong Isbell topology, the open open topology and the bounded open topology. By Theorem 2.2 if the space $C_\tau(Y,Z)$ is an element of $\mathcal{A}$, then τ is coordinately $\mathcal{A}-$jointly continuous.*

Remark 3.1. We recall that (see [3] and [11]) that the space $C_{\tau_{co}}(Y,Z)$ belongs to the family $\mathcal{A}$ in the following cases:

(1_i) $\mathcal{A}$ is the family of all T_i-spaces, $i = 0, 1, 2, 3, 3\frac{1}{2}$ and $Z \in \mathcal{A}$.

(2) $\mathcal{A}$ is the family of all topological spaces whose weight is not greater than a certain fixed infinite cardinal, Y is localy compact and $Y, Z \in \mathcal{A}$.

(3) $\mathcal{A}$ is the family of all compact spaces, Y is a discrete Tychonoff space, Z contains a subspace homeomorphic the space of real numbers with the usual topology and $Z \in \mathcal{A}$.

(4) $\mathcal{A}$ is the family of all Baire spaces (a space is a *Baire space* provided that every countable intersection of open dense subsets is dense), and Y is a locally compact, paracompact space.

(5) $\mathcal{A}$ is the family of all metrizable spaces, $Z \in \mathcal{A}$ and Y is a hemicompact space (a Hausdorff space Y is *hemicompact* if in the family of all compact subspaces of X ordered by $\subset$ there exists a countable cofinal subfamily).

Corollary 3.2. *By Corollary 3.1 in cases mentioned in the above Remark 3.1 the compact open topology on $C(Y,Z)$ is coordinately $\mathcal{A}-$jointly continuous.*

Remark 3.2. We recall that (see [10] and [11]) that the space $C_{\tau_{is}}(Y,Z)$ belongs to the family $\mathcal{A}$ in the following cases:

(1_i) $\mathcal{A}$ is the family of all T_i-spaces, $i = 0, 1, 2$ and $Z \in \mathcal{A}$.

(2) $\mathcal{A}$ is the family of all second countable spaces, Y is core-compact and $Y, Z \in \mathcal{A}$.

Corollary 3.3. *By Corollary 3.1 in cases mentioned in the above Remark 3.2 the Isbell topology on $C(Y,Z)$ is coordinately $\mathcal{A}-$jointly continuous.*

Corollary 3.4. *The compact open topology, the Isbell topology, the strong Isbell, the open-open topology and the bounded open topology on $C(Y,Z)$ are coordinately jointly continuous topologies.*

Theorem 3.2. *The pointwise topology τ_p on $C(Y,Z)$ is coordinately $\mathcal{A}-$splitting.*

Proof. Let $F : X \times Y \to Z$ be a coordinately continuous map, where X is an element of $\mathcal{A}$. We need to prove that the map $\widehat{F} : X \to C_{\tau_p}(Y,Z)$ is continuous.

Indeed, let x be an element of X and $(\{y\}, W)$ is an open neighbourhood of $\widehat{F}(x)$ in $C_{\tau_p}(Y,Z)$, where $y \in Y$ and W is an open set of Z,. Then $\widehat{F}(x)(y) = F(x,y) \in W$. Since the map $F^y : X \to Z$ is continuous there exists an open neighbourhood V_x of x in X such that $F^y(V_x) \subseteq W$. This means that $\widehat{F}(V_x) \subseteq (\{y\}, W)$, that is, $\widehat{F}$ is continuous. Thus, the topology τ_p is coordinately $\mathcal{A}-$splitting. □

Corollary 3.5. *The pointwise topology τ_p on $C(Y,Z)$ is coordinately splitting.*

Theorem 3.3. *Let $C_{\tau_p}(Y,Z) \in \mathcal{A}$. Then the pointwise topology τ_p is the greatest coordinately $\mathcal{A}-$splitting topology.*

Proof. By Theorems 2.2 and 2.3 since the topology τ_p is coordinately $\mathcal{A}-$jointly continuous and $C_{\tau_p}(Y,Z) \in \mathcal{A}$ we have that $\tau(\mathcal{A}) \subseteq \tau_p$.

On the other hand by Theorems 2.3 and 3.2, $\tau_p \subseteq \tau(\mathcal{A})$. Thus $\tau_p = \tau$ □

Corollary 3.6. *The pointwise topology τ_p on $C(Y,Z)$ is the greatest coordinately splitting topology.*

4. On the Greatest $\mathcal{A}-$splitting Topology

Theorem 4.1. *The pointwise topology τ_p is $\mathcal{A}-$jointly continuous if and only if for every space $X \in \mathcal{A}$ the coordinate continuity of the map $F : X \times Y \to Z$ implies the continuity of F.*

Proof. Suppose that the pointwise topology τ_p is $\mathcal{A}-$jointly continuous. Let X be an element of $\mathcal{A}$ such that the map $F : X \times Y \to Z$ is coordinately continuous. Since the topology τ_p is coordinately $\mathcal{A}-$splitting (see Theorem 3.2), the map $\widehat{F} \equiv G : X \to C_{\tau_p}(Y,Z)$ is continuous. On the other hand since τ_p is $\mathcal{A}-$jointly continuous the map $\widetilde{G} = F : X \times Y \to Z$ is continuous.

Conversely, suppose that for every space $X \in \mathcal{A}$ the coordinate continuity of the map $F : X \times Y \to Z$ implies the continuity of this map. Let X be an element of $\mathcal{A}$ and $G : X \to C_{\tau_p}(Y, Z)$ be a continuous map. Since the topology τ_p is coordinately $\mathcal{A}-$jointly continuous (see Theorem 3.1) the map $\widetilde{G} \equiv F : X \times Y \to Z$ is coordinately continuous. By assumption the map $\widetilde{G}$ is continuous. Thus, the topology τ_p is $\mathcal{A}-$jointly continuous. □

Corollary 4.1. *The pointwise topology τ_p is jointly continuous if and only if for every space X the coordinate continuity of the map $F : X \times Y \to Z$ implies the continuity of the map F.*

Corollary 4.2. *Let $C_{\tau_p}(Y, Z) \in \mathcal{A}$. If for every space $X \in \mathcal{A}$ the coordinate continuity of the map $F : X \times Y \to Z$ implies the continuity of F, then the topology τ_p concides with the greatest $\mathcal{A}-$splitting topology.*

Corollary 4.3. *If for every space the coordinate continuity of the map $F : X \times Y \to Z$ implies the continuity of F, then the pointwise topology τ_p coincides with the greatest splitting topology.*

Theorem 4.2. *Let Y be a corecompact space. If for every space X the coordinate continuity of the map $F : X \times Y \to Z$ implies the continuity of F, then the pointwise topology coincides with the Isbell topology.*

Proof. Since the space Y is corecompact the Isbell topology is the greatest splitting (see Introduction). On the other hand the pointwise topology is always splitting (see Introduction) and by Corollary 4.1 is also jointly continuous. Therefore, τ_p concides with the greatest splitting. Thus, $\tau_p \equiv \tau_{is}$ □

The proof of the following theorem is similar to the proof of Theorem 4.2.

Theorem 4.3. *Let Y be a localy compact space. If for every space X the coordinate continuity of the map $F : X \times Y \to Z$ implies the continuity of this map, then the pointwise topology coincides with the compact open topology.*

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